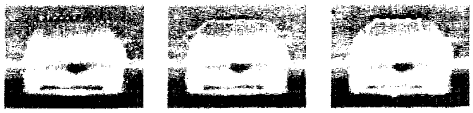

a *b* *c*

Fig. 3 *Results of infrared images containing vehicle*
a Input image and initial contour
b Proposed method
c Snake algorithm

Conclusions: We propose a new boundary extraction scheme that can be applied to a low-quality image such as an infrared image. When a training set that describes a desired object is given, it extracts boundary using a statistical shape descriptor and the combined Bayesian objective function. Experimental results show faster convergence of the objective function that PDM and the robustness of noise, occlusion, and background edges.

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Hae Chul Choi and Seong Dae Kim (*Department of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), 373-1, Kusong-dong, Yusong-gu, Taejon 305-701, Republic of Korea*)

E-mail: chc@sdvision.kaist.ac.kr

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Closed-form solutions for discriminative filtering using impulse restoration techniques

A. P. Mendonça and E. A. B. da Silva

In earlier works, the authors have dealt with the template detection problem by using two-dimensional discriminative filtering. A discriminative filter maximises, for a given template, the energy concentration in a single sample of its output. In this work, the authors frame discriminative filtering as an impulse restoration problem and propose closed-form solutions for it.

Introduction: Template detection using discriminative filtering consists of making a convolution between the image and an operator computed for a specific template. The expected output has samples with large power where the template is present and small power otherwise. It has been proposed in [1, 2] and extended for the two-dimensional case in [3].

One can give an alternative interpretation of discriminative filtering for template detection as follows: the input image $g(m,n)$ can be expressed as

$$g(m, n) = f(m - m_0, n - n_0) + b(m, n) \tag{1}$$

where $f(m - m_0, n - n_0)$ is the template centred at position (m_0,n_0) and $b(m,n)$ is the rest of the image. Equation (1) can be regarded as the result of an impulse at position (m_0,n_0) being distorted by a linear operator $f(m,n)$ and corrupted with additive noise $b(m,n)$. The ideal discriminative filter would obtain at its output a large value at position (m_0,n_0) and zero otherwise. This is nothing but an impulse; therefore, the ideal discriminative filter is the one that restores the impulse distorted by the template and corrupted with the rest of the image. In this Letter, we formulate discriminative filtering as an impulse restoration problem. Using this formulation, we find closed-form solutions for it. It is important to note that, in previous approaches [3], solutions could only be obtained by numerical optimisation.

Two-dimensional discriminative filtering: Discriminative filters maximise the energy of an output sample whenever a matching template is found. The two-dimensional discriminative signal-to-noise ratio (DSNR_2), defined in [3], is a measure that accounts not only for the maximum energy of a sample, but also considers its energy in relation to the other samples. Thus, for two-dimensional discriminative filters, we need to maximise:

$$\mathrm{DSNR}_2 = \frac{c_{i,j}^2}{\left(\sum_m \sum_n c_{m,n}^2\right) - c_{i,j}^2} \tag{2}$$

The $c_{m,n}$ coefficients are obtained after a two-dimensional convolution between an input image window $\mathbf{U} = \{u_{m,n}\}$ and a linear filter $\boldsymbol{\Theta}$ having impulse response $\theta_{m,n}$. $\boldsymbol{\Theta}$ is computed for each different template to be matched. The coefficient $c_{i,j}$ is the one where we wish to concentrate the output signal energy.

Discriminative filtering against impulse restoration: To formulate the impulse restoration problem in matrix notation [4], an image $g(m,n)$ is transformed in a column vector $\mathcal{G}(k)$ by concatenating its transposed rows. The equivalent template in the unknown position (m_0,n_0) becomes $\mathcal{F}(k - k_0)$ and the noise $b(m,n)$, corresponding to the rest of the image, becomes $\mathcal{B}(k)$. Therefore, (1) may be rewritten as

$$\mathcal{G}(k) = \mathcal{F}(k - k_0) + \mathcal{B}(k) = \mathcal{F}(k)^* \delta(k - k_0) + \mathcal{B}(k). \tag{3}$$

Considering the above convolution to be circular and the image in (1) to be of dimensions $(2T+1) \times (2T+1)$, with $-T \le m,n \le T$, we can express (1) and (2) in matrix notation as

$$\mathbf{g} = \mathbf{F}\delta + \mathbf{b} \tag{4}$$

where

$$\mathbf{g} = \begin{bmatrix} g(-T, -T) \\ g(-T, -T+1) \\ \vdots \\ g(+T, +T) \end{bmatrix} \quad \delta = \begin{bmatrix} \delta(-T, -T) \\ \delta(-T, -T+1) \\ \vdots \\ \delta(+T, +T) \end{bmatrix}$$
$$\mathbf{F} = \begin{bmatrix} \mathcal{F}(0) & \mathcal{F}(1) & \cdots & \mathcal{F}((2T+1)^2 - 1) \\ \mathcal{F}(1) & \mathcal{F}(2) & \cdots & \mathcal{F}(0) \\ \vdots & \vdots & \cdots & \vdots \\ \mathcal{F}((2T+1)^2 - 1) & \mathcal{F}((2T+1)^2) & \cdots & \mathcal{F}((2T+1)^2 - 2) \end{bmatrix} \tag{5}$$

The impulse restoration problem consists of, given $\mathbf{g}$ and $\mathbf{F}$, estimate the vector δ. We assume that the random noise vector $\mathbf{b}$ $((2T+1)^2 \times 1)$ is Gaussian, with zero mean and covariance $\mathbf{C}_\mathbf{b}$ $((2T+1)^2 \times (2T+1)^2)$, and that δ $((2T+1)^2 \times 1)$ is zero mean with covariance $\mathbf{I}$ $((2T+1)^2 \times (2T+1)^2$ identity). Using the orthogonality principle, the best linear estimate $\hat{\boldsymbol{\delta}} = \mathbf{A}\mathbf{g}$ that minimises $\mathrm{E}[\|\delta - \hat{\boldsymbol{\delta}}\|^2]$ is obtained by making the error $(\delta - \hat{\boldsymbol{\delta}})$ uncorrelated [4] to the observation $\mathbf{g}$. Also, assuming that δ and $\mathbf{b}$ are uncorrelated, we find the following result:

$$\hat{\boldsymbol{\delta}} = \mathbf{A}\mathbf{g} = \mathbf{F}^t(\mathbf{F}\mathbf{F}^t + \mathbf{C}_\mathbf{b})^{-1}\mathbf{g} \tag{6}$$

At this point, it is important to show how the matrices $\mathbf{F}$ and $\mathbf{A}$ can be derived from the original two-dimensional sequences $\mathbf{U}$ and $\boldsymbol{\Theta}$ (see text below (2)). To do this, we use a block-matrix notation. Let $\mathbf{H}_\mathbf{r}$ be the operator that transforms the row $\mathbf{r}$ of a generic window $\mathbf{v}$ $(2N+1) \times (2N+1)$ in a $(2N+1) \times (2N+1)$ circulant matrix, according to the following rule:

$$\mathbf{H}_\mathbf{r}(\mathbf{v}) = \begin{bmatrix} \mathbf{v}_{\mathbf{r},0} & \mathbf{v}_{\mathbf{r},-1} & \mathbf{v}_{\mathbf{r},-2} & \cdots & \mathbf{v}_{\mathbf{r},+3} & \mathbf{v}_{\mathbf{r},+2} & \mathbf{v}_{\mathbf{r},+1} \\ \mathbf{v}_{\mathbf{r},+1} & \mathbf{v}_{\mathbf{r},0} & \mathbf{v}_{\mathbf{r},-1} & \cdots & \mathbf{v}_{\mathbf{r},+4} & \mathbf{v}_{\mathbf{r},+3} & \mathbf{v}_{\mathbf{r},+2} \\ & \vdots & & & & \vdots & \\ \mathbf{v}_{\mathbf{r},+N} & \mathbf{v}_{\mathbf{r},+N-1} & \mathbf{v}_{\mathbf{r},+N-2} & \cdots & \mathbf{v}_{\mathbf{r},-N+2} & \mathbf{v}_{\mathbf{r},-N+1} & \mathbf{v}_{\mathbf{r},-N} \\ \mathbf{v}_{\mathbf{r},-N} & \mathbf{v}_{\mathbf{r},+N} & \mathbf{v}_{\mathbf{r},+N-1} & \cdots & \mathbf{v}_{\mathbf{r},+N+3} & \mathbf{v}_{\mathbf{r},-N+2} & \mathbf{v}_{\mathbf{r},-N+1} \\ & \vdots & & & & \vdots & \\ \mathbf{v}_{\mathbf{r},-1} & \mathbf{v}_{\mathbf{r},-2} & \mathbf{v}_{\mathbf{r},-3} & \cdots & \mathbf{v}_{\mathbf{r},+2} & \mathbf{v}_{\mathbf{r},+1} & \mathbf{v}_{\mathbf{r},0} \end{bmatrix} \tag{7}$$

With the above definition, $\mathbf{F}$ and $\mathbf{A}$ become

$$\mathbf{F} = \begin{bmatrix} \mathbf{H}_0(\mathbf{U}) & \mathbf{H}_{-1}(\mathbf{U}) & \cdots & \mathbf{H}_{-T}(\mathbf{U}) & \mathbf{H}_{+T}(\mathbf{U}) & \cdots & \mathbf{H}_{+1}(\mathbf{U}) \\ \mathbf{H}_{+1}(\mathbf{U}) & \mathbf{H}_0(\mathbf{U}) & \cdots & \mathbf{H}_{-T+1}(\mathbf{U}) & \mathbf{H}_{-T}(\mathbf{U}) & \cdots & \mathbf{H}_{+2}(\mathbf{U}) \\ & \vdots & & & \vdots & & \\ \mathbf{H}_{+T}(\mathbf{U}) & \mathbf{H}_{+T-1}(\mathbf{U}) & \cdots & \mathbf{H}_0(\mathbf{U}) & \mathbf{H}_{-1}(\mathbf{U}) & \cdots & \mathbf{H}_{-T}(\mathbf{U}) \\ \mathbf{H}_{-T}(\mathbf{U}) & \mathbf{H}_{+T}(\mathbf{U}) & \cdots & \mathbf{H}_{+1}(\mathbf{U}) & \mathbf{H}_0(\mathbf{U}) & \cdots & \mathbf{H}_{-T+1}(\mathbf{U}) \\ & \vdots & & & \vdots & & \\ \mathbf{H}_{-1}(\mathbf{U}) & \mathbf{H}_{-2}(\mathbf{U}) & \cdots & \mathbf{H}_{+T}(\mathbf{U}) & \mathbf{H}_{+T-1}(\mathbf{U}) & \cdots & \mathbf{H}_0(\mathbf{U}) \end{bmatrix} \quad (8)$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}_0(\boldsymbol{\Theta}) & \mathbf{H}_{-1}(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_{-T}(\boldsymbol{\Theta}) & \mathbf{H}_{+T}(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_{+1}(\boldsymbol{\Theta}) \\ \mathbf{H}_{+1}(\boldsymbol{\Theta}) & \mathbf{H}_0(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_{-T+1}(\boldsymbol{\Theta}) & \mathbf{H}_{-T}(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_{+2}(\boldsymbol{\Theta}) \\ & \vdots & & & \vdots & & \\ \mathbf{H}_{+T}(\boldsymbol{\Theta}) & \mathbf{H}_{+T-1}(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_0(\boldsymbol{\Theta}) & \mathbf{H}_{-1}(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_{-T}(\boldsymbol{\Theta}) \\ \mathbf{H}_{-T}(\boldsymbol{\Theta}) & \mathbf{H}_{+T}(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_{+1}(\boldsymbol{\Theta}) & \mathbf{H}_0(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_{-T+1}(\boldsymbol{\Theta}) \\ & \vdots & & & \vdots & & \\ \mathbf{H}_{-1}(\boldsymbol{\Theta}) & \mathbf{H}_{-2}(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_{+T}(\boldsymbol{\Theta}) & \mathbf{H}_{+T-1}(\boldsymbol{\Theta}) & \cdots & \mathbf{H}_0(\boldsymbol{\Theta}) \end{bmatrix} \quad (9)$$

After finding $\mathbf{A}$ from (6), we determine, by inspection, the coefficients of the two-dimensional filter $\boldsymbol{\Theta}$ from (9). Note that being $\mathbf{F}$ of the form of (8), it can be shown that (6) always gives $\mathbf{A}$ of the form of (9).

Alternative approach to impulse restoration: With the formulation of the preceding Section, we find $\boldsymbol{\Theta}$ that maximises the DSNR_2 for a given template using the impulse restoration approach (6). However, that computation does not avoid that, when noise is present, a different template offers a better DSNR_2, when filtered with $\boldsymbol{\Theta}$. An alternative solution for this problem is to consider DSNR_2 against filter $\boldsymbol{\Theta}$ and template $\mathbf{U}$ and look for a $\boldsymbol{\Theta}$ which maximises the DSNR_2 when $\mathbf{U}$ varies. This is the same as the 'alternative approach' of the discriminative filtering, described in [3]. In what follows, we model it as an impulse restoration problem.

To maximise the DNSR_2, we have to minimise $\mathrm{E}[\|\delta - \hat{\boldsymbol{\delta}}\|^2]$. The solution for the alternative approach is found after determining $\mathbf{A}$ so that $\mathrm{E}[\|\delta - \hat{\boldsymbol{\delta}}\|^2]$ (a function of $\mathbf{A}$ and $\mathbf{F}$) is minimised when $\mathbf{F}$ is the template $\underline{\mathbf{F}}$ the discrimination of which is desired. Since

$$\mathrm{E}[\|\delta - \hat{\boldsymbol{\delta}}\|^2] = \mathrm{E}\{b^t\mathbf{A}^t\mathbf{A}b\} + \mathrm{E}\{\|(\mathbf{I} - \mathbf{AF})\delta\|^2\} \quad (10)$$

only $\mathrm{E}\{\|(\mathbf{I} - \mathbf{AF})\delta\|^2\} \geq 0$ depends on $\mathbf{F}$, so the minimum error, considering $\mathbf{A}$ a constant matrix, is obtained when equality is satisfied for $\mathbf{F} = \underline{\mathbf{F}}$. For this case, the solution is $\mathbf{A} = \underline{\mathbf{F}}^{-1}$. Note that this solution does not depend on statistics of $b(m,n)$.

Mixed approach: We can mix the two formulations above, as in [3]. The mixed approach weights two terms. The first term ($\mathrm{E}[\|\delta - \hat{\boldsymbol{\delta}}\|^2]$) is such that the smaller it is, the larger is the DSNR_2. The second one ($\mathrm{E}\{\|(\mathbf{I} - \mathbf{A}\underline{\mathbf{F}})\delta\|^2\}$) is such that the smaller it is, the closer the solution is to the alternative one. It can be proved that the mixed solution (11) is similar to (6), changing $\mathbf{C}_b$ by $(1-K)\,\mathbf{C}_b$, where K $(0 \leq K \leq 1)$ is the weight of the second term and $(1-K)$ of the first term:

$$\hat{\boldsymbol{\delta}} = \mathbf{F}^t(\mathbf{F}\mathbf{F}^t + \{1-K\}\mathbf{C}_b)^{-1}\mathbf{g} \quad (11)$$

Simulation results: To simulate the proposed method, we used the same detection scheme of [3]. The chosen templates were a 45° corner of size 9×9 and the 'E' character (Arial, 8 pt). Fig. 1 shows detections of 45° corners and Fig. 2 shows 'E' detections in a text over Lena. We used $K = 1$ in (11).

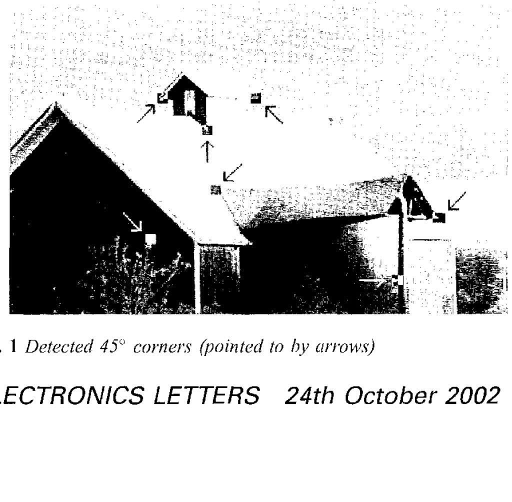

Fig. 1 *Detected 45° corners (pointed to by arrows)*

Fig. 2 *Detected 'E'*

In the proposed method, the DSNR obtained in matching conditions is of the order 0.5 (for the 45° corner). This is a significant improvement over the method in [3], that gives DSNR of the order of 0.09. In the example of detection of letter 'E', we can see that just one out of 14 letters was not detected (detection was assumed when the DSNR_2 was larger than 0.315).

Conclusions: We have presented discriminative filtering modelled as an impulse restoration problem. The main objective of the proposed method is to obtain a two-dimensional filter that, when convolved with the image template, generates as output an image with the energy concentrated in only one sample. It is important to point out that the advantage of the proposed method in relation to the one of [3] is that the computation of the discriminative filters in [3] requires the use of numerical optimisation, while the proposed method gives closed-form solutions. These are more accurate, offering a small number of false detections, unlike the method in [3]. Indeed, the simulations show that the method works very well with real images.

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A. P. Mendonça (*IME/COPPE, DE/3, Pç Gen. Tibúrcio, 80, CEP 22.290-270, Rio de Janeiro, Brazil*)

E. A. B. da Silva (*COPPE/DEL/EE/UFRJ, Cx.P. 68504, CEP. 21945-970, Rio de Janeiro, Brazil*)

E-mail: eduardo@lps.ufrj.br

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Morphological operators

M.H. Sedaaghi

The author and colleagues have already proved that mediated morphological filters (MEDMFs) remove speckle, Gaussian or salt&pepper noise better than classical morphological filters (MFs) and linear methods. They have also demonstrated the dominance of MEDMFs in a multiple noisy environment compared with MFs and linear filters. Here, they describe novel new research that has led to new morphological operators being successfully devised, the performance of which is better than MEDMFs for both single and multiple denoising. The new operators employ a special combination of weighted median filters and MFs.