intensity gradients for a video sequence also differ as (importantly) they are taken after Gaussian smoothing over a set of frames. An eigenvalue test will be more successful in the LK method because it is only necessary to establish a sound opticalflow vector and not to find the type of corner. As in the corner detector, if the test uses trace(**G**)/det(**G**), which is akin to a conditioning number, then the curvature factor can be isolated. An analysis of the LLSE solution for the OFE as an inverse problem or through an inertia tensor method also will give rise this to test. Table 1, using the synthetic sequences available by anonymous ftp to csd.uwo.ca in / pub/vision/, shows the equivalence of results, which approach a lower-bound error. Tuning of the test is required to set the acceptance density.

Conclusions: A confidence test requires an understanding of why it works. The Bayesian explanation is not fully satisfactory as it relies on *a priori* assumptions. The differential geometry explanation is helpful as it results in a test without formal calculation of eigenvalues. First differentials are sufficient to indicate curvature for this application.

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Design of wavelets with optimal correction of ringing effect

M.A.M. Rodrigues, E.A.B. da Silva and P.S.R. Diniz

The authors address the problem of designing wavelet transforms such that the synthesis wavelet has no sidelobes. To meet this criterion, constraints on the filter coefficients search space have been derived. The resulting wavelets, when used for image compression, produce minimum ringing artefacts.

Introduction: The construction of a wavelet decomposition scheme requires a suitable lowpass/highpass filter pair, or eqiuvalently a two-band filter bank. For image processing, linear phase finite impulse response (FIR) filters are preferred because of their low complexity and to avoid artefacts produced by nonlinear phase filters under quantisation [1]. Biorthogonal wavelets are appropriate because they allow perfect reconstruction in the absence of quantisation [2].

The design technique employed in this Letter is based on optimisation of the impulse response of the filters. Some previous work in this area will be mentioned briefly. In the context of subband coding, Kronander [3] aims to reduce ringing effects by generating filters having a step response with minimum overshoot. In [4], a concern with the peak-to-sidelobe ratio of the wavelets was demonstrated when selecting the best functions. In [5], it was observed that non-negative synthesis scaling functions could be used to reduce the ringing effect. It was verified that such non-negative scaling functions were generated by non-negative lowpass synthesis filters. In [6], it was verified that most of the objective performance criteria used in wavelet design have little correlation to a subjective assessment of picture quality. In fact, it was noted that the damping of the oscillations of the wavelet towards its tails plays an important role in the visibility of coding errors, especially the ringing. A measure of such damping was defined, being called the peak-to-peak ratio (*PPR*), whose value can be computed as shown in [7] or [8]. The product of the coding gain and the peak-to-peak ratio, ($G_c \times PPR$), was shown in [6] to have a high correlation (≥ 0.85) to the subjective evaluation of compressed images by human experts.

In this Letter, we generate wavelets by first imposing a condition of maximum *PPR*, i.e. $PPR = 2$, to the synthesis wavelet, which is then optimised for the coding gain. This approach differs from that presented in [7] because, in that case, the product ($G_c \times PPR$) was optimised and, in the present case, only G_c is optimised. The optimisation becomes easier because the search space is reduced when the $PPR = 2$ condition is imposed. The theoretical coding gain G_c is computed as by Katto and Yasuda [9], assuming that the input is modelled by an auto-regressive process (AR(1)), with an autocorrelation $\rho = 0.95$. Also, in the present work, we changed the way the value of *PPR* is calculated. In [7], only the three peaks on the left, nearest to the centre of the wavelet, were used in the calculation. Here, the biggest peaks are used. This change in the algorithm improves the accuracy of the *PPR* calculation.

Filter design: The filter design technique is the same as used in [7], however, the filter coefficients are constrained such that the *PPR* of the resulting wavelet will be two (the maximum value). Note that, as the *PPR* is calculated directly from the wavelet, to translate such a condition in terms of the filter coefficients is not a simple task. Eqn. 1 shows how the analysis lowpass filter is iterated:

$$H_0^{(i)}(z) = \prod_{k=0}^{i-1} H_0\left(z^{2^k}\right) \tag{1}$$

The analysis highpass iterated filter is given by

$$H_1^{(i)}(z) = H_1\left(z^{2^{i-1}}\right) H_0^{(i-1)}(z) \tag{2}$$

A piecewise constant continuous time function $\psi^{(i)}(t)$ is associated to the iterated filter:

$$\psi^{(i)}(t) = 2^{\frac{i}{2}} h_1^{(i)}(n) \qquad n/2^i \leq t < \frac{n+1}{2^i} \tag{3}$$

The analysis wavelet is obtained in the limit

$$\psi(t) = \lim_{i \to \infty} \psi^{(i)}(t)$$

For the synthesis scaling function $\tilde{\phi}(t)$ and wavelet $\tilde{\psi}(t)$, similar formulae apply, except that $H_0(z)$ and $H_1(z)$ are replaced by $G_0(z)$ and $G_1(z)$, respectively, where $G_0(z)$ is the lowpass synthesis filter and $G_1(z)$ is the highpass synthesis filter.

As can be seen in [7], in order to have $PPR = 2$, the wavelet must not oscillate beyond its first lobe. This means that the wavelet cannot change polarity between its beginning and its half point (or its point of symmetry), see Fig. 1. Observing eqns. 1 – 3, and remembering that *PPR* is calculated from the synthesis wavelet, it is straightforward to see that a *sufficient* condition is that both $g_0(n)$ and $g_1(n)$ have all coefficients of the same polarity. However, in this situation, $G_1(z)$ will not have a zero at $z = 1$ and, consequently, $H_0(z)$ and $H_1(z)$ should be chosen such that their even indexed coefficients (those multiplied by even indexed powers of z) have an inverse polarity to the odd indexed coefficients. Following the above assumption, $H_0(z)$ will have no zeros at $z = -1$, which does not yield a useful wavelet. Fortunately, this condition is not necessary, and there are many examples of filters having $PPR = 2$ that do not follow it.

In practice, most useful wavelets have a high coding gain which implies good regularity [6] and smooth appearance and, taking this into consideration, the calculation of *PPR* can be simplified. After designing a large number of filters, we reached a less restrictive condition. For maximum *PPR*, we conjecture that, if the coding gain is sufficiently high, the following conditions should be true:

(i) The lowpass synthesis filter impulse response is monotonically crescent, up to the central coefficient:

$$h_0(n+1) - h_0(n) > 0 \mid n \in \left[0, \frac{L_0 - 1}{2}\right] \qquad (4)$$

(ii) The highpass synthesis filter has an impulse response with the same polarity up to the central coefficient:

$$\mathrm{sign}(h_1(0)) = \mathrm{sign}(h_1(n)) \mid n \in \left[0, \frac{L_1 - 1}{2}\right] \qquad (5)$$

These conditions are also *sufficient*. Sufficiency was tested in more than a hundred filters for each filter size, from $L = 5$ up to $L = 14$. The filter coefficients were chosen randomly, although following the above criteria. The test consisted of verifying whether the above criteria are met in filters whose *PPR* is not maximum and where $G_c > 5$. No such filter was found. However, if there are no situations where *PPR* is maximum and the above criteria are not met, the conditions will also be *necessary*. We applied a similar test to determine this and found not a single case (we also constrained G_c to be > 5). Hence, we also conjecture that the above conditions should be *necessary* for a maximum *PPR* when the coding gain is sufficiently high.

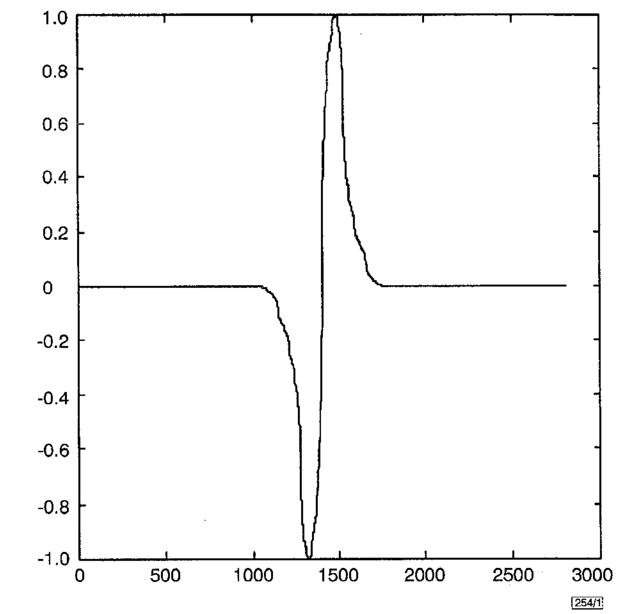

Fig. 1 *Wavelet constrained to PPR = 2 and optimised for maximum G_c*

With these easy to verify conditions, optimisation is limited only to the coding gain calculation. A simulated annealing algorithm was used mostly since it is more robust when avoiding local solutions and because the result is quite independent from the initial conditions. All filter coefficients were affected in the optimisation, see [10]. Some results are shown in Table 1 and Fig. 1.

Table 1: Performance of wavelets generated from carrying a simulated annealing multivariable optimisation using maximum PPR condition

Filter	G_c	*PPR*	$G_c \times PPR$
	dB		
[1.0 1.2×10^5 7.8×10^5]	8.99	2.00	15.832
[1.0 1.1 1417 9234]	8.99	2.00	15.831
[1.0 12.5 12.7 1×10^4 7×10^4]	8.99	2.00	15.832
[1.0 98.6 828.2 828.9 73×10^3 5×10^5	8.99	2.00	15.833

Three-stage wavelet decomposition was used. In the first column, only half of the filter coefficients are shown. Filter lengths are even

Although less restrictive than the condition of all coefficients having the same polarity, the conditions in eqns. 4 and 5 limit very much the search space. Examining only eqn. 4, note that the coefficients must be > 1 (consider, without loss of generality, that $h_0(0) = 1$), which confines the region of support to about half the range of the real numbers ($1 < h_0(1) < \infty$). Secondly, note that any new coefficient must be greater than the previous one. In two dimensions ($L_0 = 5$ or $L_0 = 6$), the region of support ($1 < h_0(1) < \infty$ and $h_0(1) < h_0(2) < \infty$) is ~ 1/8 of the total area of the real plane. Following the above reasomng, the fraction of the entire $\mathbf{R}^N$ space, for which eqn. 4 is valid, is:

$$F(N) = \frac{1}{2^{2N-1}}$$

This limitation makes the optimisation easier, and, as necessity was tested, it does not keep the algorithm from finding otherwise better results.

Conclusions: We have found a simple way to design wavelets with constrained oscillations that will yield compressed images with little ringing. The results presented here indicate that when maximum *PPR* is forced, good filters can be found. The main feature of the method is an easier calculation of the constraining conditions. Our current objective is to generate wavelet transforms with a good performance in image compression and to analyse their inherent properties. However, these wavelets with maximum *PPR* may be useful in a number of other applications.

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