Design of wavelets for image compression satisfying perceptual criteria


Indexing terms: Image coding, Wavelet transforms

The authors present a wavelet design technique for image compression based on the optimisation of a lowpass analysis filter using an index which has been shown to have high correlation to subjective judgment of image quality. The wavelets obtained show a surprisingly good performance with relatively low orders.

Introduction: The construction of a wavelet decomposition scheme requires a suitable lowpass/highpass filter pair, or equivalently a two-band filter bank. For image processing, linear phase filters are preferred because of their low complexity and to avoid artefacts produced by nonlinear phase filters under quantisation [1]. Biorthogonal wavelets are suitable for use in this case [2].

In [3], it was verified that most of the objective performance criteria used in wavelet design have little correlation to subjective assessment of picture quality. In fact, it was noted that the damping of the oscillations of the wavelet towards its tails plays an important role in the visibility of coding errors. A measure of such damping was defined, called PPR, whose value can be calculated for two different situations as shown in eqn. 1 (see Fig. 1):

\[
PPR = \frac{2(x+y)}{(x+y) + (y+z)} \quad PPR = \frac{2x}{x+y} \quad (1)
\]

The two definitions in eqn. 1 apply to wavelets with even and odd symmetry, respectively. The product of the coding gain and the PPR (\(G_x \times PPR\)) was shown in [3] to have high correlation (\(\geq 0.85\)) to the subjective evaluation of compressed images by human experts. In this work, \(G_x\) is the theoretical coding gain from Katto and Yasuda [4], assuming that the input is modelled by an autoregressive process (AR(1)), with autocorrelation \(\rho = 0.95\).

Fig. 1 Parameters used in PPR calculation for wavelets

a With even symmetry
b With odd symmetry

Filter design: Biorthogonality in a two-band filter bank is equivalent to

\[
H_0(z)H_1(-z) - H_0(-z)H_1(z) = 2z^{-2m+1} \quad (2)
\]

where \(H_0(z)\) is the analysis lowpass filter of length \(L_0\) and \(H_1(z)\) is the analysis highpass filter of length \(L_1\). The problem with optimising both \(H_0(z)\) and \(H_1(z)\) directly and simultaneously, is their highly nonlinear relationship, as shown in eqn. 2, what may create complex error surfaces, depending on the objective function used.

One possible solution to this problem was proposed in [2]: while keeping \(H_0(z)\) fixed, new polynomials \(H_1(z)\) of a higher degree than \(H_0(z)\) can be found satisfying eqn. 2. However, this result is of limited use because, when keeping \(H_0(z)\) fixed, there is a single optimisation parameter for each increase of four in the polynomial degree of \(H_1(z)\).

Conversely, it can be shown that, given \(H_0(z)\), there is only one filter \(H_1(z)\) such that \(L_1 \leq L_0\) [2]. In this case, eqn. 2 enables calculation of \(H_1(z)\) from \(H_0(z)\) by solving a set of linear equations, whenever there is a solution. Since the synthesis lowpass filter \(G_0(z)\), of length \(L_0\), and the highpass filter \(G_1(z)\), of length \(L_1\), can be determined from the pair (\(H_0(z), H_1(z)\)) [2] one can optimise the wavelet filter bank using any design criteria by simply changing the coefficients of \(H_1(z)\). When it is desired that \(L_1 \geq L_0\) the optimisation can be performed by changing the coefficients of \(H_0(z)\) or \(G_0(z)\) instead of \(H_1(z)\). This approach is general because it spans the space of filters for which \(L_0 = L_1 - 4k, k \in Z\) for even \(L_0\) and \(L_0 = L_1 - 4k - 2, k \in Z\) for odd \(L_0\).

Degenerated filters: Consider the filter bank in Table 1. \(H_0(z)\) has the ratio of the central to the first coefficient equal to 32.5. This observation raises the question of whether the first and the last coefficients of \(H_0(z)\) can be eliminated, creating a new reduced order filter bank without sacrifice of its performance. For this particular case the answer is affirmative, as can be seen from the first two entries of Table 2 (note that since \(L_0 > L_0\) a new \(H_0(z)\) has to be found from eqn. 2). This indicates that the original filter bank can be considered as a degenerated case of the new one. The degenerated case is used here in the sense that at least one degree of freedom is redundant (e.g. a straight line is a degenerated
parabola). The filters are symmetrical and have an even length, which means that a zero at \( z = -1 \) is guaranteed. In the case of filters having an odd length the zero 'moves away' from \( z = -1 \) in the process above, so they will not yield regular wavelets.

### Table 2: Performance of wavelets generated from filter pair \( K \) of Table 1 and its reduced length versions

| \( L_o/L_i \) | \( G_i [\text{dB}] \) | \( \text{PPR} \) | \( G_i \times \text{PPR} \) |
|---------------|----------------|------------|----------------|----------------|
| 6/10          | 9.59           | 1.65       | 15.06          |
| 8/8           | 9.33           | 1.85       | 17.53          |
| 6/6           | 9.29           | 1.88       | 17.69          |
| 4/4           | 8.80           | 2.00       | 17.60          |

3-stage wavelet decomposition was used

Table 2 shows that, if the order reduction process is continued, wavelets generated by the reduced order filter banks will also be good up to length 6. The \( G_i \) is somewhat reduced, but the PPR compensates and keeps wavelet quality. It is important to note, though, that this order reduction process does not always generate good wavelets. In fact, no correlation was found between the ratio of the central to the first coefficients of a filter and the possibility of reducing the order of the filter bank without affecting the performance of its corresponding wavelet. However, such behaviour raises the question of whether one can generate better filter banks by degenerating (in the sense described above) good ones.

### Table 3: Performance of wavelets generated from Haar filters and its increased length versions

| Length | Opt. \( k \) | \( G_i [\text{dB}] \) | \( \text{PPR} \) | \( G_i \times \text{PPR} \) |
|--------|-------------|----------------|------------|----------------|----------------|
| 2      | —           | 7.92           | 2.00       | 15.84          |
| 4      | —           | 8.99           | 2.00       | 17.98          |
| 6      | —           | 9.69           | 1.97       | 19.56          |
| 8      | —           | 9.16           | 2.00       | 18.32          |
| 10     | —           | 9.16           | 2.00       | 18.32          |
| 12     | —           | 9.16           | 2.00       | 18.32          |
| 6 op.  | —           | 9.34           | 1.94       | 18.32          |

3-stage wavelet decomposition was used and \( k \) is optimised multiplier, as used in equs. 3 and 4. Last filter bank is optimum among filter banks of length 6

Generating good wavelets from Haar filter banks: When reducing the order of a filter bank, some information (a degree of freedom) is lost. Therefore, when increasing the order of (i.e. degenerating) a filter bank, some optimisation is possible, as shown below. We begin with the Haar biorthogonal filter pair \( \{2(1+z^{-1}), 2(1-z^{-1})\} \). The performance of this wavelet is modest, see Table 3. However, after the first increase in size, we have the class of filters of length 4, for which \( H(z) \) is

\[
\alpha [1 + \frac{k(z^{-1} + z^{-2}) + z^{-3}}{2}]
\]

where the \( \alpha \)s are used herein as normalisation factors. The maximum of the product \( G_i \times \text{PPR} \) is found when \( k = -6.49 \). Proceeding further, we have the class of filters of length 6, for which \( H(z) \) is

\[
\alpha [1 + \frac{k(z^{-1} - 6.49z^{-2} - 6.49z^{-3} + z^{-4}) + z^{-5}}{2}]
\]

The maximum is found for \( k = 9.6903 \). Note that the process does not lead to optimum filter banks. However, it has been noted that the above filter bank and the optimum filter for length 6 are very close, not only in the value of \( G_i \times \text{PPR} \) (see Table 3), but also in the sense that, starting from the first one, the second filter bank is achieved by a simple gradient search algorithm over the surface of the function \( G_i \times \text{PPR} \).

Proceeding in the same way, we can increase filter bank order by optimising a single variable function. After some iterations (as the filter bank order increases), the variation in the index \( G_i \times \text{PPR} \) is very small, placing a practical limit in this process at length 8.

**Conclusions:** The performance achieved with the wavelets generated from these filter banks, even those of small order, is quite impressive and superior to most known wavelets, see Table 4. Furthermore, there is good indication that we can use these filter banks to initialise a multi-variable optimisation and achieve even better results.

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References


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Error robustness improvement of video codes with two-way decodable codes

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**Indexed terms:** Video codes

Owing to the variable rate nature of compressed video bit streams, a single error may cause loss of synchronisation which results in false decoding. Therefore, when the decoder detects an error, it usually skips all the bits until the next synchronisation word. In this case, the correct bits located between the corrupted bit and the synchronisation word cannot be decoded without using two-way decodable codes which permit reverse decoding of the bit stream. The authors explain the two-way decoding concept and use it to modify the ITU H.263 video coding algorithm in order to improve its robustness against channel errors.

**Introduction:** The latest advances in video compression technology have opened the way to low bit-rate mobile audiovisual communications. It is now possible to encode video signals at as low as 10 kbit/s thanks to the new ITU video coding standard, H.263 [1].

High compression ratios can be achieved by removing the redundant information from the video signal which makes compressed bit streams more sensitive to errors. This high sensitivity is the main problem for mobile communications due to the high error rates of mobile networks.

The existing video coding standards have been recommended mainly for fixed network applications, and therefore do not use any error resilience tools which can cope with the error rates of...