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Wavelet transform image coding using lattice vector quantisation

D.G. Sampson, E.A.B. da Silva and M. Ghanbari

Indexing terms: Image coding, Vector quantisation, Wavelet transforms

A novel method for the efficient coding of image wavelet coefficients using zerotree multistage lattice vector quantisation is presented. This method achieves high compression ratios with good picture quality, maintaining a very simple implementation. Simulation results demonstrate that the coding performance of this algorithm favourably compares to some of the best reported image compression results.

Introduction: Recently, wavelet transforms have been employed for image and video compression applications. Several investigations have been reported regarding the development of efficient methods for coding wavelet coefficients. The most successful of these methods take into consideration the structural similarity between bands of the same orientation by generating zerotree roots. Shapiro [1] has proposed a very efficient coding method referred to as the embedded zerotree wavelet algorithm (EZW), where wavelet coefficients are coded using zerotree successive-approximation arithmetic-coded scalar quantisation. The EZW coder produced some of the best image compression results reported so far.

It has been shown that vector quantisation can offer better coding performance compared to scalar quantisation, mainly due to its inherent advantage of fractional bit allocation among the vector components [2]. This motivates the investigation on the development of efficient wavelet coding algorithms based on vector quantisation. Lattice vector quantisation (LVQ) is a type of VQ where the codebook is designed based on regular lattices. A regular lattice is a discrete set of points in k-dimensional space, which can be generated by the integral linear combination of a given set of basis vectors [3]. The main advantage of LVQ is the significant reduction in the encoding complexity, typically required by full-search VQ [4]. This is accomplished by exploiting the structural properties of regular lattices.

This Letter contributes a novel method for wavelet transform coding using zerotree multistage lattice vector quantisation. Simulation results demonstrate that this method outperforms EZW, and it offers 2–3dB improvement over the standard JPEG coder.

Wavelet transform coding using lattice vector quantisation: A new wavelet transform coder has been developed. The coding algorithm is outlined in Fig. 1. In this method, blocks of the most significant coefficients are coded using a zerotree multistage lattice vector quantiser. At each stage, the residual quantisation error of the previous passes is further refined, until a certain level of distortion is achieved, or the bit rate budget is exhausted. This provides the means to guarantee that an arbitrary level of average distortion for each band is met (noise shaping), which is convenient for performing bit allocation among the wavelet coefficients taking into consideration the human visual system (HVS) sensitivity to each frequency band [5]. It also offers control over the maximum level of quantisation error made in each wavelet coefficient. This is important for reducing unpleasant artefacts in picture quality, because the error introduced by a poorly quantised single wavelet coefficient can be spread over an area of the reconstructed image [6]. Furthermore, similarities among the bands of the same orientation are exploited by producing zerotree roots, so that a single symbol is used to indicate that a wavelet coefficient and all its corresponding ones are zero [1].

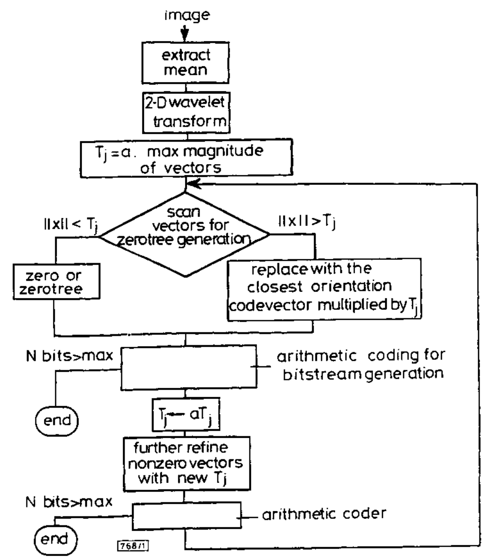

Fig. 1 *Flow chart of coding algorithm*

A key element of the proposed algorithm is the multistage lattice vector quantisation. The LVQ algorithm developed in this Letter offers the advantage that only a limited number of lattice codevectors are used, so that they can be efficiently encoded using an adaptive arithmetic coder. Hence, we can avoid the obvious difficulties of designing an efficient entropy coder for a very large lattice codebook, which is typically required in most LVQ methods. Next, we describe the principles of this method. A given input vector $\mathbf{x}$ is coded with a series of vectors of decreasing magnitudes $\{\|\mathbf{y}_j\| = a^j\|\mathbf{x}\|; j = 1, 2, \ldots\}$, where, j is the number of coding stages, $a < 1$ is the approximation scaling factor, $\|\mathbf{y}_j\|$ is the magnitude of the reconstruction codevector at stage j, and $\|\mathbf{x}\|$ is the magnitude of the input vector. At each stage, the orientation of the reconstruction vector is selected from a finite set of unit energy codevectors, referred as the orientation codebook, $\hat{\mathbf{Y}} = \{\hat{\mathbf{y}}_i: \|\hat{\mathbf{y}}_i\| = 1; i = 1, 2, \ldots, N\}$, which remains the same at each stage. Fig. 2 illustrates this process. After n stages, the reconstructed vector is formed as

$$\mathbf{Q}_n(\mathbf{x}) = \{a\|\mathbf{x}\|\hat{\mathbf{y}}_1 + a^2\|\mathbf{x}\|\hat{\mathbf{y}}_2 + a^3\|\mathbf{x}\|\hat{\mathbf{y}}_3 + \cdots + a^n\|\mathbf{x}\|\hat{\mathbf{y}}_n\}$$

To guarantee that the residual error $\{\mathbf{x} - \mathbf{Q}_n(\mathbf{x})\}$ at stage n converges into zero as $n \to \infty$, some constraints need to be imposed to the design of the orientation codebook. It was found that, the number of stages required for almost perfect reconstruction of the original vector depends on the values of : θ_{max}, which is the maximum angle between any possible input vector and its closest available orientation codevector, and the approximation scaling factor a.

To achieve higher compression ratios, the number of stages required to obtain a certain distortion level must be minimum. Because the minimum a required for convergence increases with θ_{max} and the number of stages increases with a, it is implied that the selected orientation codebook must be designed so that θ_{max} is as small as possible. Nevertheless, there is a compromise between the value of θ_{max} and the resolution of the orientation codebook, determined by $\log_2 N/k$, where N is the codebook population and k is the vector dimension. In the scalar case ($k = 1$), $\theta_{max} = 0°$. However, larger gains in bit rate reduction can be achieved, when higher dimensional vectors are used, despite the larger values of θ_{max}, due to the fractional bit allocation obtained by vector quantisation. The orientation codebooks have been designed based on different regular lattices. We have found that the first shells of the lattices D_4, E_8 and Λ_{16}, which are known to give the best sphere packing properties at dimensions 4, 8, and 16, respectively [3], offer the best performance, because they provide the best tradeoff between the population and θ_{max}.

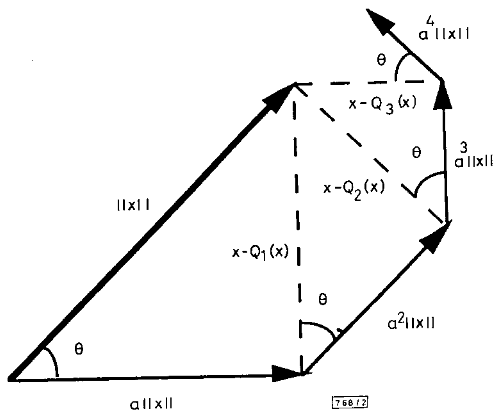

Fig. 2 *Successive approximation using vectors*

Simulation results and conclusions: The performance of the coder introduced in this Letter has been tested for various lattice codebooks and compared with the standard JPEG coder and the embedded zerotree wavelet image coder (EZW) proposed by Shapiro [1]. Table 1 summarises the peak signal to noise ratio (PSNR) results obtained by these methods for coding the test image Lena with resolution 256 × 256 × 8 and the set of five ISO/CCITT test images with resolution 720 × 576 × 8, at a bit rate of 0.4bit/pixel. In Table 1, the orientation codebooks are built from the first

Table 1: PSNR performance of the proposed method for several test images at 0.4bit/pixel compared with EZW and JPEG

Test image	D_4	E_8	L_{16}	EZW	JPEG
Barbara	29.36	30.60	30.90	29.03	27.27
Boats	34.19	34.78	35.24	34.29	32.63
Girl	35.27	35.91	36.12	35.14	33.98
Gold	31.01	32.76	32.61	32.48	31.38
Zelda	38.43	39.36	39.44	39.08	37.16
Lena 256	30.13	30.15	30.29	30.06	28.07

spherical shell of the regular lattices D_4, E_8 and Λ_{16}. In general, although there are no substantial differences in the performance of the three codebooks, higher dimensional codebooks result in better PSNR values and the best performance is always achieved by using the Λ_{16}-based codebook. These simulation results also demonstrate that the proposed coding scheme achieves considerably better R-D performance as compared with the JPEG coder. The improvement in PSNR over the JPEG-coded images is constantly around 2.50dB for Λ_{16} and 1.50dB for the D_4 codebook. Finally, comparisons with the EZW image coder [1], which is a very efficient coder employing zerotree scalar quantisation of wavelet coefficients, favour the proposed zerotree lattice vector quantiser.

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Method of lines for analysis of arbitrarily curved waveguide bends

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Indexing terms: Method of lines, Optical waveguide theory

The analysis of arbitrarily curved optical waveguide bends is described with the method of lines.

Introduction: Optical waveguide bends are part of many circuits, e.g. in connecting lines of couplers, power dividers.

Analysis: In this Section we will give the basic equations for the modelling of waveguide bends. We start with the bend in Fig. 1. We divide the bend into sectors with suitable angles α_m. Each sector is approximated by a straight waveguide with end planes equal to the sector planes, which are not normal to the propagation direction. Therefore the propagation steps are individual on each discretisation line. The basic element is shown in Fig. 1*b*. The MoL-BPM uses not only guided but also radiation modes in the structure. Therefore the fields in the two optical waveguides I and II including reflections are given by

$$\bar{\phi}^I = e^{-\Gamma_I \bar{z}}\mathbf{A}^I + e^{\Gamma_I \bar{z}}\mathbf{B}^I \quad (1)$$

$$\bar{\phi}^{II} = e^{-\Gamma_{II} \bar{z}'}\mathbf{A}^{II} + e^{\Gamma_{II} \bar{z}'}\mathbf{B}^{II} \quad (2)$$

For all quantities a subscript *e* or *h* for the TE or TM mode case, respectively, must be added. The components of the vectors A^I and A^{II} are the amplitudes of the modes in the $+z$ and $+z'$ direction, and the components of B^I and B^{II} are the amplitudes in the $-z$ and $-z'$ direction. Γ is the diagonal matrix of the propagation constants.

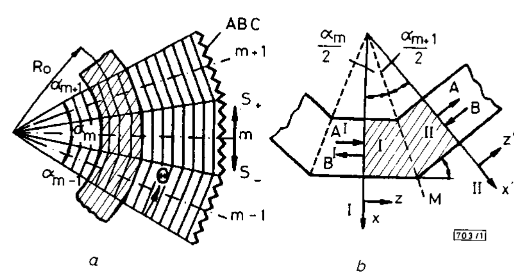

Fig. 1 *Model of waveguide bend and its basic element*

a Model of waveguide bend
b Basic element

The next step is the field matching in cross-section *M* in which each point has another distance to the reference planes I and II. The distances are equal to the individual propagation steps Δz. The main components which have to be matched are

$$\text{TM}: E_y = \frac{\partial}{\partial \bar{y}}\left(\epsilon_r^{-1}\frac{\partial}{\partial \bar{y}}\phi_h\right) + \phi_h \quad \eta_0 H_x = -j\frac{\partial \phi_h}{\partial \bar{z}} \quad (3)$$

$$\text{TE}: E_x = \frac{\partial}{\partial \bar{x}}\left(\epsilon_r^{-1}\frac{\partial}{\partial \bar{x}}\phi_e\right) + \phi_e \quad \eta_0 H_y = j\frac{\partial \phi_e}{\partial \bar{z}} \quad (4)$$

Because the two waveguide cross sections are identical instead of the field components the potential and its derivative according to *z* and *z'*, respectively, are to be matched. From eqns. 1 and 2 we