Seismic wavelets and earth's impulse response amplitude and phase recovery using second order statistics or deterministic methods

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SUMMARY

In this paper we approach the problem of recovering the earth's impulse response and the seismic wavelets used to probe the earth by the use of techniques inspired in blind equalization methods developed for digital communication systems. We present a recently proposed class of algorithms to recover, by seismogram observation only, both amplitude and phase of the seismic wavelets and the earth's impulse response, assuming a single-input multiple-output (SIMO) finite impulse response (FIR) model. In this class of algorithms no assumptions regarding minimum phase of the seismic wavelets are necessary and the earth's impulse response can be assumed both white or colored (using second order statistics) or even deterministic (using deterministic methods). The only assumption on the seismic wavelets used is that there should be no common zeros on their Z-transforms. With just this assumption we have shown that the recovery of both amplitude and phase of the earth's impulse response and the seismic wavelets themselves can be accomplished using only second order statistics or direct algebraic manipulations of the seismograms. The only minor drawback is a mild scalar ambiguity intrinsic to the problem. Tests are presented confirming the validity of these algorithms. The results obtained are promising, indicating that this class of algorithms has the potential of opening avenues for a large number of new applications.

INTRODUCTION

SIMO FIR systems have been the subject of intense investigations on the area of digital communications since (Tong et al., 1991) and (Tong et al., 1994), where it was first shown that by using only second order statistics of the outputs, it is possible to recover both the system and its input, as long as the input signal is white stationary and the system's impulse responses do not share common zeros on their Z-transforms. (Moulines et al., 1994) and (Moulines et al., 1995) later extended their approach allowing colored stationary inputs by employing a subspace method (SSM). Enhancements were further carried out by (Slock, 1994) and (Meraim et al., 1995) where linear prediction (LP) was first applied to the SIMO case. (Ding, 1996) and Ding (1997) presented another interesting statistical algorithm based on an outer product decomposition (OPD). (Gesbert and Duhamel, 1997) presented a multi-step linear prediction (MSLP) approach to further enhance the LP algorithm providing better estimators based on a larger second order statistics' window. Finally (Tsatsanis and Xu, 1997) presented a somewhat robust algorithm allowing colored input, thus enhancing the SSM algorithm.

A different approach using a deterministic model assuming negligible additive noise has also emerged. First explored by (Liu et al., 1994) through the subchannel matching (SCM) algorithm, it was shown that an overdetermined system could be constructed to recover both the system and its input by a simple system matching concept. Afterwards a more robust method based on least squares smoothing (LSS) was presented by (Tong and Zhao, 1998) and (Zhao and Tong, 1999) with later enhancements by (Tong and Zhao, 1999). Finally a series of oblique projection (OP) based algorithms were presented by (Yu and Tong, 2001).

The main motivations for all of these papers, both statistical and deterministic, were related to the problems derived by using higher order statistics (or minimum entropy) for the single-input single-output (SISO) deconvolution problem, which has intrinsic convergence problems (Ding and Li, 2001). The purpose of this abstract is to present the application of the algorithms developed on the previously cited papers to the seismic processing environment, establishing the first link to this exciting new field and clarifying some misconceptions engraved on the geophysics community. It should be pointed out, however, that other algorithms for both the statistical and deterministic cases exist, addressing different aspects of the problem, but for the sake of brevity these were the only ones implemented and validated.

PROBLEM FORMULATION

The notations $(\cdot)^\dagger$, $(\cdot)^T$, $(\cdot)^{(K)}$ and $\mathbf{0}_{m,n}$ stand respectively for the Moore-Penrose pseudo-inverse, the transpose of a matrix, an index K and an $m \times n$ zero matrix. The SIMO FIR system model with L outputs is described by

$$x_l(k) = \sum_{m=-\infty}^{\infty} s(m)h_l(k-m) + n_l(k), \quad l \in \{1,2,\ldots,L\}, \qquad (1)$$

where $s(k)$ is the input signal, $h_l(k)$ is the l-th system impulse response with length M ($h_l(k) = 0$ for $k < 0$ and $k \geq M$) and $x_l(k)$ is the l-th output signal. Translating this to the seismic problem we have that $s(k)$ is the earth's impulse response, $h_l(k)$ is the l-th seismic wavelet (which may also incorporate the receiver effect) used to probe the earth and $x_l(k)$ is the seismogram obtained by the l-th seismic wavelet. Define the $K+M-1 \times 1$ input vector by $\mathbf{s}^{(K+M-1)}(k) \triangleq [s(k), s(k-1), \ldots, s(k-K-M+2)]^T$, the $KL \times K+M-1$ block toeplitz filtering matrix $\mathbf{H}^{(K)}$ by

$$\mathbf{H}^{(K)} \triangleq \begin{bmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(M-1) & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \mathbf{h}(0) & \cdots & \mathbf{h}(M-1) & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & & & & & \ddots & \vdots & & \vdots \\ 0 & \cdots & \cdots & 0 & 0 & \cdots & \mathbf{h}(0) & \cdots & \mathbf{h}(M-1) \end{bmatrix} \qquad (2)$$

with $\mathbf{h}(k) \triangleq [h_1(k),\, h_2(k),\, \ldots,\, h_L(k)]^T$, the $KL \times 1$ output vector by $\mathbf{x}^{(K)}(k) \triangleq [\mathbf{x}^T(k), \mathbf{x}^T(k-1), \ldots, \mathbf{x}^T(k-K+1)]^T$ with $\mathbf{x}(k) \triangleq [x_1(k), x_2(k), \ldots, x_L(k)]^T$, and finally the $KL \times 1$ additive noise vector by $\mathbf{n}^{(K)}(k) \triangleq [\mathbf{n}^T(k), \mathbf{n}^T(k-1), \ldots, \mathbf{n}^T(k-K+1)]^T$ with $\mathbf{n}(k) \triangleq [n_1(k), n_2(k), \ldots, n_L(k)]^T$. The same model of (1) may be expressed in a matrix format by

$$\mathbf{x}^{(K)}(k) = \mathbf{H}^{(K)}\mathbf{s}^{(K+M-1)}(k) + \mathbf{n}^{(K)}(k). \qquad (3)$$

Our problem is then to find the earth's impulse response $s(k)$ and the seismic wavelets $\mathbf{h}(k)$ based only on the measured seismograms $\mathbf{x}(k)$.

PERFECT RECONSTRUCTION MISO FIR INVERSE SYSTEM

To find a multiple-input single-output (MISO) FIR inverse system to our SIMO FIR system we may take a simple approach. Assuming that *no additive noise is present* and assuming that $\mathbf{H}^{(K)}$ *is left invertible by the* $M+K-1 \times KL$ *matrix* $\mathbf{F}^{(K)}$, we may write

$$\mathbf{F}^{(K)}\mathbf{x}^{(K)}(k) = \mathbf{F}^{(K)}\mathbf{H}^{(K)}\mathbf{s}^{(K+M-1)}(k) = \mathbf{s}^{(K+M-1)}(k). \qquad (4)$$

Representing a perfect reconstruction (PR) MISO FIR inverse system with delay i and impulse response length K by $\mathbf{f}^{(i,K)}(k) \triangleq [f_1^{(i,K)}(k), f_2^{(i,K)}(k), \ldots, f_L^{(i,K)}(k)]^T$ (i.e. $s(k-i) = \sum_{l=1}^{L}\sum_n f_l^{(i,K)}(k-n)\sum_m s(m)\, h_l(n-m)$), the $(i+1)$-th line of $\mathbf{F}^{(K)}$, $\mathbf{f}_i^{(K)}$, is then a MISO FIR inverse system given by

$$\mathbf{f}_i^{(K)} \triangleq [\mathbf{f}^{(i,K)T}(0),\, \mathbf{f}^{(i,K)T}(1),\, \ldots,\, \mathbf{f}^{(i,K)T}(K-1)]. \qquad (5)$$

This implies that if $\mathbf{H}^{(K)}$ has a left inverse, then there exists K PR MISO FIR inverse systems of length K that recover $s(k)$ perfectly. This is a major difference if compared to the SISO case.

RESTRICTIONS RELATED TO THE INVERTIBILITY OF $\mathbf{H}^{(K)}$

The following theorem (Serpedin and Giannakis, 1999) relates the left invetibility of $\mathbf{H}^{(K)}$, the full column rank condition, to the Z-transforms of the seismic wavelets $\mathbf{h}(k)$.

Theorem 1. *The polynomials with maximum degree $M-1$ given by $\dot{h}_l(z)$, for $l = 1,\ldots,L$, do not share common zeros on their Z-transforms (i.e. $\dot{\mathbf{h}}(z) \triangleq [\dot{h}_1(z),\, \dot{h}_2(z),\, \ldots,\, \dot{h}_L(z)]^T \triangleq \sum_{k=0}^{M-1} \mathbf{h}(k) z^{-k} \neq 0,\ \forall z$. where $\mathbf{h}(k) = 0,\ k > M-1$ and $k < 0$) if and only if $\mathbf{H}^{(K)}$, a $KL \times K+M-1$ block toeplitz matrix given by* (2)*, has full column rank (is left invertible) for all $K \geq M-1$.*

The direct implication is that if K is chosen greater than or equal to $M-1$ and the seismic wavelets do not share common zeros, then there exists a PR MISO FIR inverse system of length K to our SIMO FIR system in (1). This is radically different from the SISO case that requires a minimum phase seismic wavelet (minimum phase FIR system) to have the PR property.

SECOND ORDER STATISTICS

Lets now define the second order statistics of our problem. Assuming that the earth's impulse response $s(k)$ and the additive noise $\mathbf{n}(k)$ are stationary, zero-mean and independent we may easily write

$$\mathbf{R}_{\mathbf{x}}^{(K)}(m) = \mathbf{H}^{(K)} \mathbf{R}_{\mathbf{s}}^{(K+M-1)}(m) \mathbf{H}^{(K)T} + \mathbf{R}_{\mathbf{n}}^{(K)}(m). \tag{6}$$

where the $K+M-1 \times K+M-1$ block toeplitz autocorrelation matrix of $\mathbf{s}^{(K+M-1)}(k)$, for a delay m, is given by

$$\mathbf{R}_{\mathbf{s}}^{(K+M-1)}(m) \triangleq \mathrm{E}[\mathbf{s}^{(K+M-1)}(k)\mathbf{s}^{(K+M-1)T}(k-m)], \tag{7}$$

the $KL \times KL$ block toeplitz autocorrelation matrix of $\mathbf{x}^{(K)}(k)$, for a delay m, is given by

$$\mathbf{R}_{\mathbf{x}}^{(K)}(m) \triangleq \mathrm{E}[\mathbf{x}^{(K)}(k)\mathbf{x}^{(K)T}(k-m)] \tag{8}$$

and $\mathbf{R}_{\mathbf{n}}^{(K)}(m)$ is similarly defined.

But so far no solutions based on the system's output (the seismograms) only were given. These will be presented on the next two sections as simple and direct pseudo-code. For further clarifications please refer to the references cited therein.

MSLP ALGORITHM

The MSLP algorithm as described here was originally developed by (Gesbert and Duhamel, 1997). It assumes that the earth's impulse response $s(k)$ is a white stationary input process uncorrelated from a white stationary noise process $\mathbf{n}(k)$, and that the seismic wavelets do not share common zeros ($\mathbf{H}^{(K)}$ is full column rank for $K \geq M-1$). As inputs the method requires: M, an overestimate of the impulse response length; K, which forms the window $(K+M+1)$ to be used for the second order statistics and has also to be chosen greater than or equal to M; an estimate of the second order statistics of the seismograms, $\mathbf{R}_{\mathbf{x}}^{(1)}(m)$ for $m = 0,1,\ldots,K+M$; and an estimate of the variance $\sigma_{\mathbf{n}}^2$ of the additive noise. For simplicity it is possible to choose $K = M$. As output estimates the algorithm provides the seismic wavelets $\mathbf{h}(k)$ and a MISO FIR inverse system $\mathbf{f}^{(K-1,K+M)}(k)$, both with a constant ambiguity factor. One special and important property is that *for perfect statistics both estimates are perfect up to a constant scale factor*. The steps for the algorithm are:

1. Estimate $\mathbf{R}_{\mathbf{x}}^{(1)}(m)$ for $m = 0,1,\ldots,K+M$;
2. Compute the $(K-i+1)L \times L$ matrices $\mathbf{P}_i$ for $i \in \{1,2,\ldots,M\}$, given by
$$\mathbf{P}_i = \left[\mathbf{R}_{\mathbf{x}}^{(K-i)}(0)\right]^{-1} \left[\mathbf{R}_{\mathbf{x}}^{(1)T}(i),\, \mathbf{R}_{\mathbf{x}}^{(1)T}(i+1),\, \ldots,\, \mathbf{R}_{\mathbf{x}}^{(1)T}(K)\right]^T; \tag{9}$$
3. Compute the $(K+1)L \times L$ matrices $\mathbf{W}_i$ for $i \in \{1,2,\ldots,M\}$, given by $\mathbf{W}_i = [\mathbf{I},\, \mathbf{0}_{L\times(M-i+1)L},\, -\mathbf{P}_i^T]^T$;
4. Compute the $ML \times (K+M)L$ matrix $\mathbf{W}$ given by
$$\mathbf{W} = \begin{bmatrix} [\mathbf{0}_{L\times(M-1)L} \quad \mathbf{W}_1^T] \\ [\mathbf{0}_{L\times(M-2)L} \quad (\mathbf{W}_2-\mathbf{W}_1)^T \quad \mathbf{0}_{L\times L}] \\ \vdots \\ [\mathbf{0}_{L\times(M-i)L} \quad (\mathbf{W}_i-\mathbf{W}_{i-1})^T \quad \mathbf{0}_{L\times(i-1)L}] \\ \vdots \\ [(\mathbf{W}_M-\mathbf{W}_{M-1})^T \quad \mathbf{0}_{L\times ML}] \end{bmatrix}, \tag{10}$$
where i refers to i-th line of $\mathbf{W}$;
5. Compute $\mathbf{R}_z$, given by $\mathbf{R}_{\mathbf{z}} = \mathbf{W}\,\mathbf{R}_{\mathbf{x}}^{(K+M)}(0)\,\mathbf{W}^T$;
6. Compute the estimate of the seismic wavelets $\mathbf{h}^{(M)} = [\mathbf{h}^T(M-1),\, \mathbf{h}^T(M-2),\, \ldots,\, \mathbf{h}^T(0)]^T$, the eigenvector associated to the greatest eigenvalue of $\tilde{\mathbf{R}}_{\mathbf{z}}^{(M)} = \mathbf{R}_{\mathbf{z}}^{(M)} - \sigma_{\mathbf{n}}^2 \mathbf{W}\mathbf{W}^T$;
7. Compute the estimate of a MISO FIR inverse system $\mathbf{f}^{(K-1,K+M)}$ by solving
$$\mathbf{u}^{(M)} = \arg\max_{\mathbf{u}^{(M)}} \frac{\mathbf{u}^{(M)T}\mathbf{R}_{\mathbf{z}}^{(M)}\mathbf{u}^{(M)}}{\mathbf{u}^{(M)T}\sigma_{\mathbf{n}}^2\mathbf{W}\mathbf{W}^T\mathbf{u}^{(M)}}, \tag{11}$$
or
$$\mathbf{u}^{(M)} = \arg\max_{\mathbf{u}^{(M)}} \frac{\mathbf{u}^{(M)T}\mathbf{R}_{\mathbf{z}}^{(M)}\mathbf{u}^{(M)}}{\mathbf{u}^{(M)T}\mathbf{u}^{(M)}} \tag{12}$$
and finally doing $\mathbf{f}^{(K-1,K+M)} = \mathbf{W}^T\mathbf{u}^{(M)}$. By using (11) we have the MMSE (unstable) solution and by using (12) we have the maximum output power (MOP) sub-optimal but stable solution.

JLSS ALGORITHM

The JLSS algorithm as described here was originally developed by (Tong and Zhao, 1999). It uses a deterministic model based on a least squares smoothing approach. It assumes that the input $s(k)$ has linear complexity (Zhao and Tong, 1999) greater than $3l+M$, a property that in practice is always true for any earth's impulse response, and that the seismic wavelets do not share common zeros. It does not assume any statistical model. Colored earth's impulse response can be recovered as well as any non-stationary earth's impulse response that respects the linear complexity hypothesis. In fact, if no additive noise is present the method recovers the seismic wavelets and the earth's impulse response perfectly up to a constant factor. As inputs it requires: $l+1$, an overestimate of the seismic wavelets' length; and the seismogram samples $\mathbf{x}(1)$ to $\mathbf{x}(N)$, where N has to be greater than $6l+3(M-1)$, M being the actual seismic wavelets' length. As output the algorithm estimates the seismic wavelets (with a constant ambiguity factor), which can then be used with (4) and (5) to create an inverse system that is able to recover the earth's impulse response. The steps for the algorithm are:

1. Construct directly from the seismograms the $L(l+1) \times N-3l$ block toeplitz actual data matrix $\mathbf{A}_l$ and the $2lL \times N-3l$ future-past data matrix $\mathbf{Z}_l$, given by
$$\mathbf{A}_l = \begin{bmatrix} \mathbf{x}(2l+1) & \mathbf{x}(2l+2) & \cdots & \mathbf{x}(N-l) \\ \mathbf{x}(2l) & \mathbf{x}(2l+1) & & \mathbf{x}(N-l-1) \\ \vdots & & \ddots & \vdots \\ \mathbf{x}(l+1) & \mathbf{x}(l+2) & \cdots & \mathbf{x}(N-2l) \end{bmatrix} \tag{13}$$

and

$$\mathbf{Z}_l = \left[\begin{array}{cccc} \mathbf{x}(3l{+}1) & \mathbf{x}(3l{+}2) & \cdots & \mathbf{x}(N) \\ \mathbf{x}(3l) & \mathbf{x}(3l{+}1) & \cdots & \mathbf{x}(N{-}1) \\ \vdots & & \ddots & \vdots \\ \mathbf{x}(2l{+}2) & \mathbf{x}(2l{+}3) & \cdots & \mathbf{x}(N{-}l{+}1) \\ \hline \mathbf{x}(l) & \mathbf{x}(l{+}1) & \cdots & \mathbf{x}(N{-}2l{-}1) \\ \mathbf{x}(l{-}1) & \mathbf{x}(l) & \cdots & \mathbf{x}(N{-}2l{-}2) \\ \vdots & & \ddots & \vdots \\ \mathbf{x}(1) & \mathbf{x}(2) & \cdots & \mathbf{x}(N{-}3l) \end{array} \right]. \quad (14)$$

2. Construct an orthogonal base of line vectors $\{\mathbf{u}_1, \ldots, \mathbf{u}_{4l}\}$ that spans the signal line space of $\mathbf{Z}_l$, with dimension $4l$. This can be acomplished by computing the QR decomposition of $\mathbf{Z}_l^T$ ($\mathbf{Z}_l^T \mathbf{E}_\mathbf{Z} = \mathbf{Q}_\mathbf{Z} \mathbf{R}_\mathbf{Z}$) and selecting the first $4l$ lines of $\mathbf{Q}_\mathbf{Z}^T$ as the signal line space of $\mathbf{Z}_l$. Note that numerical errors may appear if a QR algorithm with pivotal treatment is not used (by the use of the permutation matrix $\mathbf{E}_\mathbf{Z}$).

3. Compute the $L(l+1) \times N-3l$ matrix $\mathbf{E}_l$ which gives the projection error of the lines of the actual data matrix $\mathbf{A}_l$ into the space spanned by $\mathbf{u}_1, \ldots, \mathbf{u}_{4l}$. This is given by

$$\mathbf{E}_l = \mathbf{A}_l - \mathbf{A}_l \mathbf{U}^H \mathbf{U}, \quad \mathbf{U} = [\mathbf{u}_1^T,\ \mathbf{u}_2^T,\ \ldots,\ \mathbf{u}_{4l}^T]^T; \quad (15)$$

4. For $1 \leq m < l$, with m being the seismic wavelets' length estimate, let $\mathbf{Q}$ be the matrix whose lines are the $(L-1)(l+1)+m$ left singular vectors associated to the last $(L-1)(l+1)+m$ singular values of $\mathbf{E}_{l,l}$. Construct the block Hankel matrix

$$\mathbf{T}^{(m)} = \begin{bmatrix} \mathbf{Q}_0 & \cdots & \mathbf{Q}_{m-1} & \mathbf{Q}_m \\ \mathbf{Q}_1 & \cdots & \mathbf{Q}_m & \mathbf{Q}_{m+1} \\ \vdots & & & \vdots \\ \mathbf{Q}_{l-m} & \cdots & \mathbf{Q}_{l-1} & \mathbf{Q}_l \end{bmatrix}, \quad (16)$$

where the $(L-1)(l+1)+m \times L$ matrices $\mathbf{Q}_p$ for $0 \leq p \leq l$ are taken from $\mathbf{Q}$ using $\mathbf{Q} = [\mathbf{Q}_0,\ \mathbf{Q}_1,\ \ldots \mathbf{Q}_l]$;

5. Estimate the seismic wavelets by

$$\{M, \mathbf{h}^{(M)}\} = \arg \min_{m, \|\mathbf{h}^{(m)}\|=1} \|\mathbf{T}^{(m)} \mathbf{h}^{(m)}\|^2. \quad (17)$$

The above equation is solved, for $1 \leq m < l$, by finding the right singular vector $\mathbf{h}^{(m)}$ associated to the smallest singular value of $\mathbf{T}^{(m)}$. The seimic wavelets estimate $\mathbf{h}^{(M)} = [\mathbf{h}^T(M-1),\ \mathbf{h}^T(M-2),\ \ldots,\ \mathbf{h}^T(0)]^T$ will be given by $\mathbf{h}^{(m)}$ such that $\|\mathbf{T}^{(m)} \mathbf{h}^{(m)}\|^2$ is minimum. It is also possible, however, to choose m manually.

EXPERIMENTAL RESULTS

Two experiments are presented. Normalized root mean square error (NRMSE) was used to measure estimates quality and signal-to-noise ratio (SNR) to measure signal quality. SNR for SIMO FIR systems is given by SNR $= \mathrm{E}[\sum_{l=1}^{L}(s(k) * h_l(k))^2] /\ \mathrm{E}[\sum_{l=1}^{L} n_l^2(k)]$. NRMSE for $\hat{\mathbf{H}}^{(1)}$, the estimate of $\mathbf{H}^{(1)}$, is given by

$$\mathrm{NRMSE}_{\hat{\mathbf{H}}^{(1)}} = \min_{\alpha,i} \|\alpha[\mathbf{0}_{L,M-1},\ \hat{\mathbf{H}}_\mathrm{n},\ \mathbf{0}_{L,M-1}] - [\mathbf{0}_{L,i},\ \mathbf{H}_\mathrm{n},\ \mathbf{0}_{L,M+\hat{M}-2-i}]\| \quad (18)$$

where α is a real number correcting the constant multiplicative ambiguity, $i \in \{0, \ldots, M+\hat{M}-2\}$, $\mathbf{H}_\mathrm{n}$ is $\mathbf{H}^{(1)}$ with its lines normalized (i.e. $\sum_k h_i^2(k) = 1$ for $i = 1, \ldots, L$) and $\hat{\mathbf{H}}_\mathrm{n}$ is an estimate of $\mathbf{H}_\mathrm{n}$. NRMSE for $\hat{s}(k)$, the estimate of $s(k)$, is given by

$$\mathrm{NRMSE}_\mathbf{s} = \min_{\alpha,i} \frac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} \|\alpha \hat{s}(k-i) - s(k)\|, \quad (19)$$

where α is a real number correcting the constant multiplicative ambiguity and i is the delay of the estimate.

Experiment 1 used the system defined by

$$\mathbf{H}^{(1)} = \begin{bmatrix} 0.301654 & -0.603308 & -0.301654 & 0.150827 & 0.150827 \\ 0.301654 & -0.301654 & 0 & 0.301654 & 0.301654 \\ 0.301654 & -0.301654 & -0.301654 & -0.301654 & -0.150827 \\ 0.301654 & -0.301654 & 0.301654 & 0.120661 & 0.060330 \end{bmatrix}, \quad (20)$$

a well behaved and simple system with no common zeros. The input was made white stationary and white noise was added to the system output. The algorithms received exact estimates of both M and $\sigma_\mathbf{n}^2$ and 200 output samples. 100 runs of the experiment were carried out for each SNR tested. All algorithms cited in the introduction were implemented and tested. The mean results are shown in figures 1 and 2. All statistical algorithms converged to good results before achieving a plateau when SNR was high, due to deficiencies on the second order statistics estimation. All deterministic algorithms had almost perfect convergence when SNR was extremely high, with no plateau at all. Although not shown on 1 and 2, when no additive noise is present the deterministic algorithms converge perfectly. It should be stressed, however, that when an under- or overestimate of M is used, many of the algorithms do not converge at all. As will be shown on experiment 2, this is not the case with the MSLP algorithm.

Experiment 2 used real seismic wavelets recorded from two totally different seismic experiments. An earth's impulse response sampled at 4 ms (figure 6) and obtained by proper manipulations of a vertical seismic profile was used. Only statistical algorithms were tested and for comparison purposes a Wiener SISO spiking deconvolution algorithm was used, with later stacking of both $s(k)$ estimates to obtain the final estimate. The seismic wavelets' length M applied to the algorithms was 30 (60 ms). Greater lengths were also tested but produced poorer results for all algorithms. Since only the additive noise was statistical, 10 runs of the experiment were carried out for each SNR used. Figure 5 shows the superiority of the MSLP algorithm over the Wiener SISO algorithm for estimating the seismic wavelets. Figure 4 shows some improvement when we compare the LP and the MSLP algorithm with the Wiener SISO spiking deconvolution. Figure 3 shows estimates of the seismic wavelets for one run of the experiment at SNR=18 dB. MSLP is clearly better.

CONCLUSIONS

This abstract proved that, differently from common belief, it is possible to recover blindly both phase and amplitude of both the earth's impulse response and the seismic wavelets used to probe the earth, up to a constant multiplication factor, with only second order statistics of the output of a SIMO FIR model. For this to be possible it is assumed that no common zeros are shared by the seismic wavelets used, a much weaker assumption than the minimum phase conditions necessary for Wiener SISO algorithms. Although not shown here, both white and colored stationary earth's impulse responses, together with the seismic wavelets, can be recovered using only second order statistics of the SIMO FIR system output (Tsatsanis and Xu, 1997). Deterministic algorithms go even further allowing deterministic earth's impulse responses to be recovered, assuming no statistical hypothesis at all. Just a minor linear complexity assumption is necessary. If no additive noise is present, deterministic methods are able to recover the earth's impulse response and the seismic wavelets perfectly, up to a constant multiplication factor, with a finite sample of the seismograms only. If compared to SISO or SIMO higher order statistics (or minimum entropy) algorithms, convergence is *much* faster for both the second order statistics approach and the deterministic approach. Finally, when the seismic wavelets are known and respect the no common zeros hypothesis, a MISO FIR inverse system always exists. This means that even non-minimum phase seismic wavelets can be inverted by finite filters if they are part of a SIMO FIR model with no common zeros.

Amplitude and phase recovery

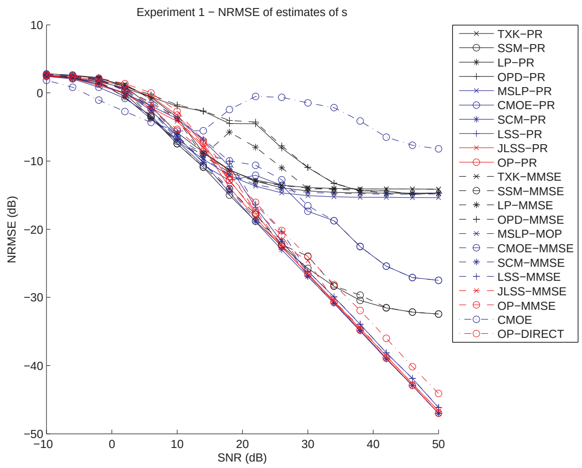

Figure 1: Experiment 1. Mean results for estimates of the input $s(k)$.

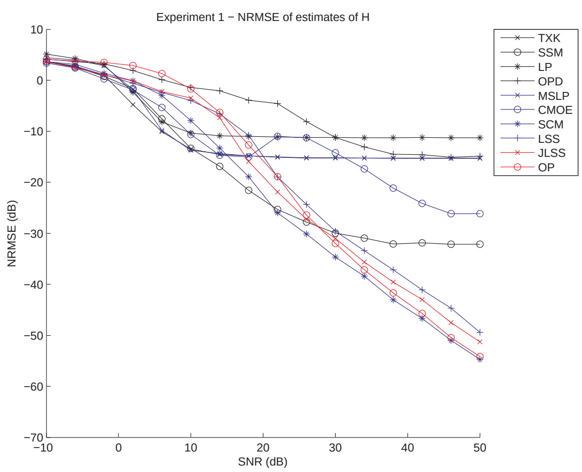

Figure 2: Experiment 1. Mean results for estimates of the system impulse response $\mathbf{H}^{(1)}$.

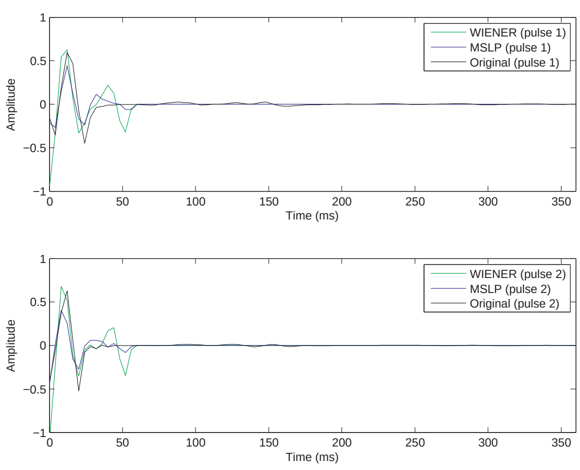

Figure 3: Experiment 2. Estimates of $\mathbf{H}^{(1)}$ obtained at SNR=18 dB, for MSLP and Wiener SISO algorithms, compared with the original seismic wavelets $\mathbf{H}^{(1)}$.

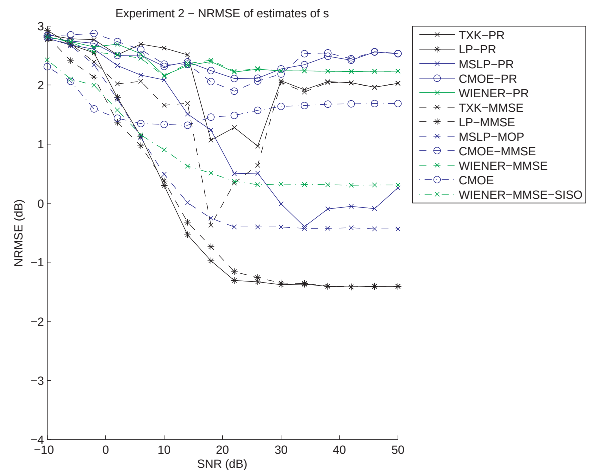

Figure 4: Experiment 2. Mean results for estimates of the earth's impulse response $s(k)$.

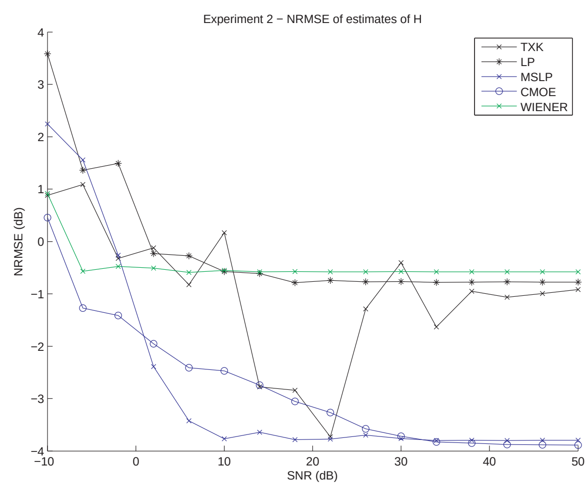

Figure 5: Experiment 2. Mean results for estimates of the seismic wavelets $\mathbf{H}^{(1)}$.

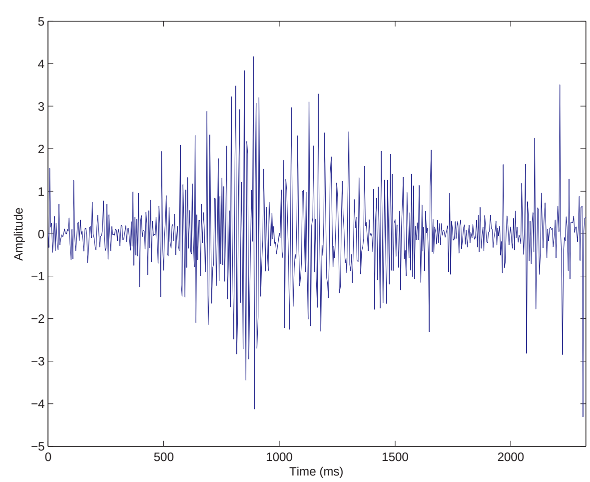

Figure 6: Experiment 2. Earth's impulse response used, sampled at 4 ms.

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