Compression of Atomic Decompositions Using R-D Optimum Dictionary Selection

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***Abstract*— Atomic decompositions have been increasingly used as signal compression tools. In general, these decompositions are obtained using a single dictionary. One may use instead several dictionaries to decompose the signal, and transmit side information in order to indicate the dictionary employed. This allows the selection of the dictionary leading to the best rate-distortion compromise. Such a scenario is encountered when decompositions that use dictionaries composed of parameterized atoms are to be encoded. In such framework, distinct quantizers applied to parameters of the atoms lead to different dictionaries. In this work, we propose a strategy, based on a training stage, to select the parameter quantizers that give near-optimum rate-distortion performance. The proposed strategy is assessed in the framework of electric power system disturbance signals compression. Simulation results show that the proposed scheme indeed achieves near-optimum R-D performance with low computational complexity.**

***Index Terms*— Compression, Atomic decompositions, Dictionaries, Rate-distortion optimization, Electric power system disturbance signals.**

I. INTRODUCTION

Recently, atomic decompositions have become important tools for the compression of several signal sources such as video [1], audio [2], biomedical (ECG) [3] and electric-power system disturbances [4]. These decompositions are adaptive approximation of signals based on a redundant dictionary. The approximation is adaptive since the functions involved, called atoms, are selected from the dictionary according to the signal being decomposed. This adaption relies on the redundancy of the dictionary which is formed by more functions than necessary to expand the signal space. The use of highly redundant dictionaries enables appropriate decompositions of a wide range of signal features, leading to efficient signal representations. The choice of the dictionary is crucial to obtain physically interpretable representations. That is, the atoms of the dictionary have to match the underlying phenomena of the signal. Several decomposition methods have been used to obtain these representations such as the method of frames [5], basis pursuit [6] and matching pursuit [7].

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Generally, the compression of atomic decompositions consists of obtaining first the approximation of the signal $\mathbf{x}$. This approximation corresponds to a linear combination of atoms selected from a single redundant dictionary D and is expressed as:

$$\hat{\mathbf{x}} = \sum_{n=0}^{M-1} \alpha_n \mathbf{g}_{\gamma(n)}. \tag{1}$$

The atoms $\mathbf{g}_{\gamma(n)}$ are indexed by the mapping $\gamma(n)$ that is defined as $\gamma : \mathbb{Z}^+ \rightarrow [1, \ldots, C_D]$; C_D is the dictionary cardinality – the number of elements in D, thus $\gamma(n) \in [1, \ldots, C_D]$; α_n denotes the coefficient, that is, the weight of $\mathbf{g}_{\gamma(n)}$; and M is the number of atoms used to approximate $\mathbf{x}$. Then, one encodes the coefficients and atom indices that are transmitted or stored. In the decoder, to reconstruct the signal one employs the same dictionary used in the encoder. Such a scheme is illustrated in Fig. 1. Note that there is no need to encode any information related to the dictionary employed since the decoder knows it beforehand. In this case, the optimum rate-distortion tradeoff is achieved by finding a compromise between the number of expansion elements and the quantization of each coefficient [8].

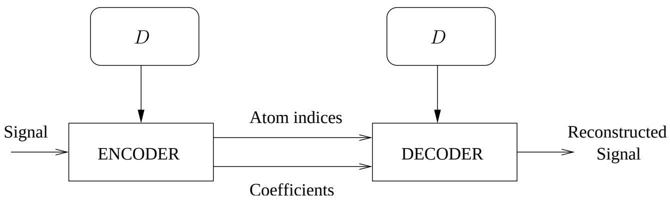


Fig. 1. Compression framework using a single redundant dictionary D.

Now suppose that instead of using a single dictionary one considers a set of redundant dictionaries described as $\mathcal{D} = \{D_i\}_{i=1,\ldots,I}$, where I is the number of dictionaries included in $\mathcal{D}$. This framework is illustrated in Fig. 2. Let each dictionary $D_i = \{\mathbf{g}_{\gamma(k)}^{D_i}\}_{k=1,\ldots,C_{D_i}}$, where $\mathbf{g}_{\gamma(k)}^{D_i}$ denotes an atom $\in D_i$ and C_{D_i} is the cardinality of D_i. In this scheme, the encoder chooses the dictionary to be used in the atomic decomposition. Considering D_i as the chosen dictionary, the atomic decomposition provides the following signal approximation:

$$\hat{\mathbf{x}}_{D_i} = \sum_{n=0}^{M_{D_i}-1} \alpha_n^{D_i} \mathbf{g}_{\gamma(n)}^{D_i}. \tag{2}$$

where M_{D_i} is the number of atoms selected from the dictionary D_i to form the signal representation $\hat{\mathbf{x}}_{D_i}$. Each dictionary yields a different signal representation. Then, one encodes the coefficients, the indices of the atoms and the side information specifying the dictionary chosen for the signal decomposition. The decoder, based on the side information, selects from $\mathcal{D}$ the dictionary used by the encoder and reconstructs the signal. The optimum rate-distortion performance is obtained when one finds the trade-off between the bits spent on side information, atom indices and coefficients that leads to the minimum distortion. This leads to an optimization problem with high computational demands and difficult to solve, since one may deal with a large set of dictionaries D_i.

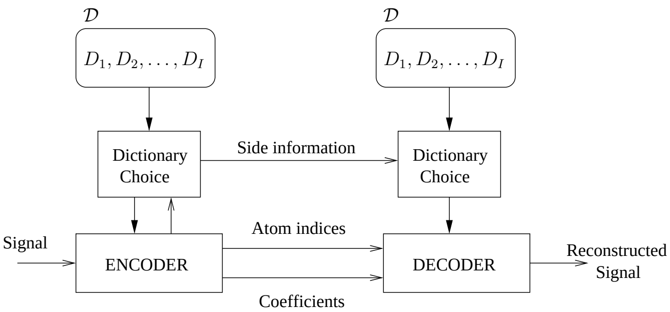


Fig. 2. Compression framework using a set $\mathcal{D}$ of redundant dictionaries.

In this paper, we address the optimization for the particular case of decompositions on parameterized dictionaries. These dictionaries are composed by atoms whose actual waveforms are given as function of a set of parameters. The several dictionaries D_i are defined by distinct quantization of the parameter space. As for example depicts Fig 3. In Fig 3, the dictionaries D_i are indexed by the amount of bits allocated to each parameter. Initially, the signal approximation composed by atoms with continuous parameters is obtained through the Matching Pursuit (MP) algorithm. The MP, introduced by Mallat and Zhang [7], is a greedy adaptive algorithm that derives the signal approximation iteratively. Then, the encoder quantizes both the coefficients and the atom parameters and encodes the side information containing the specifications of the parameter quantizers, such as dynamic range of the parameters and the amount of bits allocated to each parameter. The side information defines the dictionary.

Considering this framework, we propose a strategy based on selecting, using a training set, a reduced set of dictionaries (parameter quantizers) belonging to the convex hull of its rate-distortion characteristics. In other words, we find a set of dictionaries that may provide a good rate-distortion performance. This procedure highlights the important role of the training stage, since a dictionary may be suitable for one class of signals but not for other classes. We assess the proposed method in the compression of electric power system disturbance atomic signal decompositions presented in [4]. The results show that the proposed strategy allows to achieve near optimal rate-distortion performance with low computational complexity.

This paper is organized as follows. Section II outlines the atomic decompositions using parameterized dictionaries.

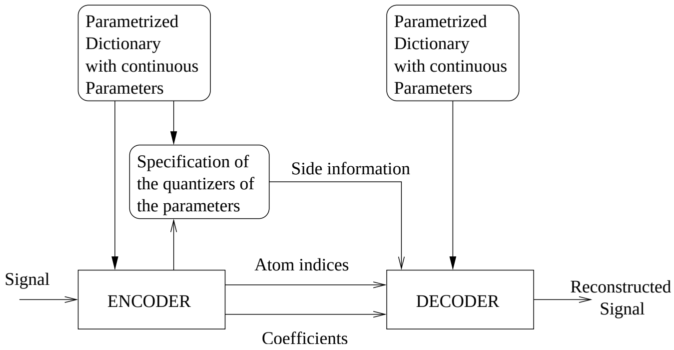


Fig. 3. Compression framework using a set of parameterized redundant dictionaries.

It also describes the practical approach to obtain these decompositions using several dictionaries defined by distinct quantizers of the parameter space. Section III explains the rate-distortion optimization based on the selection of the coefficient quantizer and the dictionary which yields minimum distortion for a given rate, and describes the strategy used to reduce its computational complexity. Section IV presents some experimental results of the proposed method in the framework of electric power system disturbance signals. Section V presents the conclusions.

II. Atomic Decompositions with Parameterized Dictionaries

Parameterized dictionaries are defined as a set $D = \{\mathbf{g}_{\gamma(k)}\}_{\gamma(k)\in\Gamma}$, such that $\|\mathbf{g}_{\gamma(k)}\| = 1$, wherein Γ is the parameter space – the set of allowed $\gamma(k)$, and the mapping $\gamma(k)$ is defined as $\gamma : \mathbb{Z}^+ \rightarrow \mathbb{R}^T$. The parameter set is $\gamma(k) = (\nu_k^t)_{t=1,\ldots,T}$, where ν_k^t denotes a parameter value and T is the amount of parameters that define $\mathbf{g}_{\gamma(k)}$. In general, these dictionaries are composed by sampling real atoms generated by scaling, translating and modulating a single window function $g(t)$ resulting in [7]:

$$g_\gamma(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) cos(\xi t + \phi) \qquad (3)$$

where s denotes the scale, ξ is the frequency modulation, u denotes the translation and ϕ is the phase, so $\gamma(k) = (s_k, \xi_k, u_k)$.

In principle, the compression of atomic decompositions using parameterized redundant dictionaries appears to be a hard task, since the parameter space Γ is infinite. Compression can only be achieved if one selects a countable finite subset $\Gamma_{D_i} \in \Gamma$ to encode the signal. Each of these subsets defines a different dictionary (see Fig. 4).

A practical approach to handle this problem consists of performing the atomic decomposition using a parameterized dictionary with continuous parameters through MP algorithm [4], [7]. Then, indexes are placed to identify the several D_i by the atom parameter quantization. This is depicted in Fig. 3 and was described in general terms in section I. The MP performs successive approximations of the signal iteratively over the elements of a redundant dictionary. At the first iteration, the MP algorithm chooses the atom with

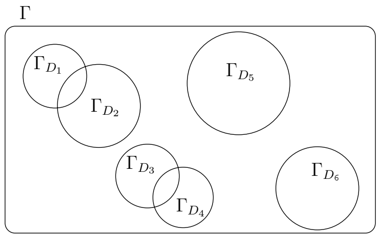


Fig. 4. The problem of compressing atomic decompositions using parameterized dictionaries.

the highest correlation with respect to the signal. The chosen atom is then scaled and subtracted from the signal obtaining a new residue. The process is repeated until the residue energy becomes sufficiently small or another stopping criterion is met. In the case of continuous parameter dictionary, on each step of the decomposition, we first search for the most correlated atom in a finite dictionary with pre-sampled parameter space, then one looks for the γ which maximizes the matching between the atom and the signal [4]. In the end of the decomposition, we obtain the sequence of pairs $(\alpha_n, \gamma(n))$, $n = 0, \ldots, M-1$ which is also known as the structure book [7]. For compression, the structure book may be quantized and one way is to quantize α_n and each parameter of $\gamma(n)$ by using a uniform scalar quantizer as [9]:

$$x_q = I_x \times \Delta_{q(x)}, \quad \text{where } I_x = \left\lfloor \frac{x + \frac{\Delta_{q(x)}}{2}}{\Delta_{q(x)}} \right\rfloor \tag{4}$$

where x is any parameter, x_q denotes its quantized version, $\Delta_{q(x)}$ denotes the quantization step and I_x is the index assigned to x. The parameters are quantized according to a dynamic range defined by their respective maximum and minimum values among all the elements of the structure book, that is:

$$\Delta_{q(x)} = \frac{x_{\max} - x_{\min}}{2^{b_x} - 1}, \tag{5}$$

where b_x is the number of bits allocated to x. The dynamic ranges and the number of bits allocated to the parameters are encoded as side information. Although this is not the case in this work, other types of quantizer could be used, besides the uniform quantizer. The quantization of the parameters and the coefficients results in the following signal approximation:

$$\hat{\mathbf{x}} = \sum_{n=0}^{M-1} Q^{\alpha}[\alpha_n] \mathbf{g}_{Q^i[\gamma(n)]}. \tag{6}$$

where $Q^{\alpha}[\cdot]$ is the quantization function of the coefficients and $Q^i[\cdot]$ denotes the quantization function associated to the dictionary D_i. Depending on the amount of bits allocated among the parameters, different atoms compound the signal approximation, that is, one uses distinct dictionaries. More precisely, the dictionary D_i used is defined by the function $Q^i[\gamma(n)] : \mathbb{R}^T \rightarrow \mathbb{R}^T$. Then the R-D optimization problem is to find the quantizers $Q^{\alpha}[\cdot]$ and $Q^i[\cdot]$ that lead to the minimum distortion for given rate.

It should be pointed out that the compression framework presented strongly differs from other compression systems using MP. The compression systems based on MP, presented so far in the related literature, achieve signal compression by quantizing just the coefficients. Here, the compression is achieved by quantizing the parameters of the structure book and the atoms used to rebuild the signal are different than the ones obtained by the decomposition algorithm. As we have discussed this compression scheme is equivalent to the use of multiple dictionaries followed by the selection of one of them for the coding of a given signal.

III. R-D Optimum Dictionary Selection

The purpose of the rate-distortion optimization is to achieve the best signal reproduction for a desired compression target [10]. In the framework at hand, one has to find a compromise between the number of atoms in the signal representation, the quantization of each coefficient, and the choice of the dictionary $D_i \in \mathcal{D}$ that is defined by the quantizers of the atom parameters. First, define the number of bits for a given atom as:

$$r = r_{\alpha} + r_{\nu^1} + \ldots + r_{\nu^t} + \ldots + r_{\nu^T}, \tag{7}$$

where r_{α} denotes the number of bits allocated to the coefficients α and r_{ν^t} is the amount of bits allocated to the parameter ν^t, such that $\gamma = \{\nu^t\}_{t=1,\ldots,T}$, and T is the number of the atom parameters. Note that one also has to determine the number N of terms in the decomposition. The total number of bits spent will be $r \times N$. The distortion of the atom is expressed as a function of the bitrates of the coefficient and the parameters resulting in:

$$d = f(r_{\alpha}, r_{\nu^1}, \ldots, r_{\nu^T}). \tag{8}$$

Consider the quantizer defined by (4) and the individual wordlengths contained in the $(T+1)$-tuple $\mathbf{b}_k = (r_{\alpha}, r_{\nu^1}, \ldots, r_{\nu^T}) \in \mathcal{B}$, where $\mathcal{B}$ denotes the set of all possible bitrate combinations allowed within the interval defined for each element of $\mathbf{b}_k$, $k = [1, 2, \ldots, K_{\mathcal{B}}]$, where $K_{\mathcal{B}}$ is the number of elements in $\mathcal{B}$. Note that each $\mathbf{b}_k$ defines a coefficient quantizer and a quantizer for the atoms' parameters. The parameter quantizer corresponds to a choice of dictionary D_i (see Eq. (2)). In order to accomplish the best rate-distortion trade-off, one should search for the $\mathbf{b}_k$ that minimizes the total distortion inserted by the encoding process given a bit-budget r_{budget}. The solution is obtained by solving the following optimization problem [10]:

$$\min_{\mathbf{b}_k \in \mathcal{B}} d_S = N \times d \quad \text{subject to } r_{\text{budget}} = N \times r. \tag{9}$$

The classical solution for this problem is based on the Lagrangian optimization [10], which corresponds to minimizing the following cost function:

$$J = d_S + \lambda\, r_{\text{budget}}, \tag{10}$$

where $\lambda \geq 0$ denotes the Lagrangian multiplier. The basic idea of this technique is depicted in Fig. 5. For a given λ, one finds the pair $(d_S^{opt}, r_{\text{budget}}^{opt})$ where J is minimum, i.e., the point where the plane wave corresponding to J hits first. The several optimum points related to different λ's form the operational rate-distortion curve.

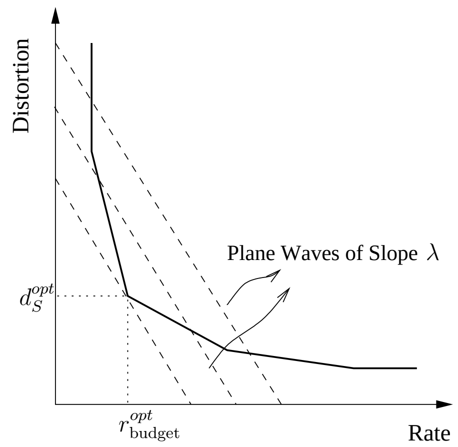


Fig. 5. Graphical interpretation of the Lagrangian optimization.

As there is no closed form for d_S with respect to $(r_\alpha, r_{\nu^1}, \ldots, r_{\nu^T})$, we had to use the approach to obtain these operational curves described in the sequel [10]. For each $\mathbf{b}_k = (r_\alpha, r_{\nu^1}, \ldots, r_{\nu^T})$, and for a given signal, compute the rate-distortion pair (r_k, d_k), yielding an R-D plot as illustrated in Fig. 6. The operational curve is obtained by connecting the points belonging to the convex hull of the region defined by the R-D pairs generated for each $\mathbf{b}_k \in \mathcal{B}$. Note in Fig. 6 that, for instance, the point B is certainly worse than A, because it presents identical distortion, but has larger rate. Likewise, the point C is also worse than A, because it presents a larger distortion for the same rate. Thus, we select the point belonging to the convex hull that for a desired compression rate gives the dictionary and coefficient quantizer which yields minimum distortion.

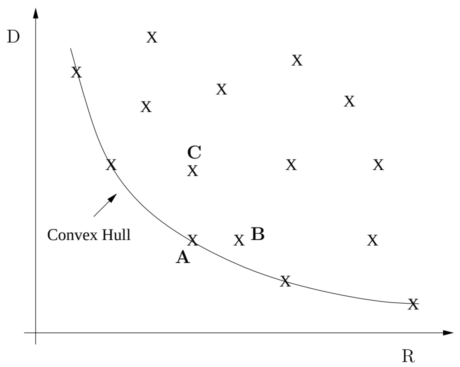


Fig. 6. Convex hull comprising the rate-distortion optimal points.

Observe that the set of R-D pairs differs from signal to signal, that is, one $\mathbf{b}_k$ found to be optimum for one signal may not be optimum for another. Thereby, we propose a strategy to diminish the size of $\mathcal{B}$ by using only the $\mathbf{b}_k$ which provide an R-D optimum dictionary and coefficient quantizer for at least one signal. We construct the operational curves for a training set of signals, from these curves we obtain a reduced set of quantizers formed by the $\mathbf{b}_k$ associated to the curves R-D optimum points. Note that this strategy is equivalent to the reduction of the number of dictionaries that can be chosen (see Figs. 2 and 3) to decompose signals. The proposed strategy aims to reduce the computational cost of the rate-distortion optimization by diminishing the total number of R-D pairs computed. From these pairs, one obtains for each signal the convex hull. In the next section, we assess the performance of the proposed approach by compressing a set of power system disturbance signals.

IV. Experimental Results

In this section, we evaluate the proposed compression framework using R-D optimum dictionary selection when applied to electric power system's disturbance signal decompositions presented in [4]. These decompositions are obtained according to the practical approach described in section II which employs the MP algorithm based on parameterized dictionaries with continuous parameters. In this decomposition algorithm, some heuristics are introduced inside the MP loop aiming at achieving physically interpretable representations according to the intrinsic phenomena of disturbance signals. In the application at hand, one uses the dictionary of exponential atoms which are defined as:

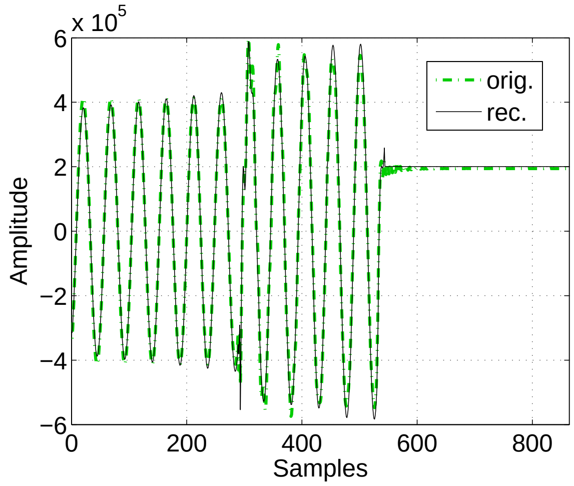
$$g_\gamma(k) = K_\gamma e^{-\rho(k-m_s)} \cos(\xi k + \phi) \quad [u(k-m_s) - u(k-m_e)] \tag{11}$$

where $u(\cdot)$ is the unit step function, K_γ is set so that $\|\mathbf{g}_\gamma\| = 1$ and $k = 1, \ldots, N_s$, in which N_s is the atom length. Thus, $\gamma(n) = (\rho_n, \xi_n, \phi_n, m_n^s, m_n^e)$, where ρ_n is the decaying factor, ξ_n denotes the frequency, ϕ_n denotes the phase, m_n^s and m_n^e are the starting and ending samples.

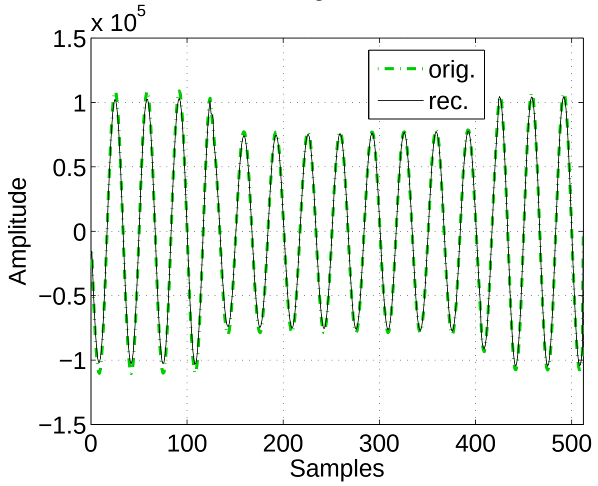
Considering atoms as in (11), the number of bits per atom is $r = r_\alpha + r_\rho + r_\xi + r_\phi + r_{m^s} + r_{m^e}$ and the distortion per atom is given by $d = f(r_\alpha, r_\rho, r_\phi)$, because r_{m^s} and r_{m^e} are defined by N_s and r_ξ is defined by the sampling frequency and the fundamental frequency in which the power system operates [4]. Therefore, $\mathbf{b}_k = (r_\alpha, r_\rho, r_\xi, r_\phi, r_{m^s}, r_{m^e})$, where r_α, r_ρ, r_ϕ are the free variables and the R-D optimization problem represented by (9) is solved as described in Section III.

First, we define $\mathcal{B}$ by setting the intervals for r_α varying from 3 to 16 bits and for r_ρ and r_ϕ, from 1 to 12 bits; all possible combinations result in $\mathcal{B}$ having 2061 elements. With the use of this set, we obtained the operational R-D curves for a training set consisted of 29 disturbance signals acquired from the Brazilian power system monitoring. This training set is characterized by the main inherent phenomena existing in electric power system's disturbance signals [11]. The amount of signals in this set is appropriate to achieve a reduced set of $\mathbf{b}_k$ that is capable of obtaining near optimum R-D performance as we see next through the results. It was observed that, from the complete set of 2061 possible $\mathbf{b}_k$ used to obtain the training set operational curves, only 292 were actually optimal for at least one signal. The use of reduced set yielded a seven-fold reduction in complexity.

Fig. 7 illustrates two disturbance signals, s_1 and s_2, that are originally represented using 16 bits per sample and do not belong to the training set. Through several tests, we verified that when they are compressed with rate values greater than 1.85 and 0.84 bits per sample, respectively, the difference between the original and reconstructed versions is unnoticeable by visual inspection. In Fig. 7, we can also observe their reconstructed versions compressed with 0.3321 and 0.4790 bits per sample, respectively. In both cases, some amplitude and phase distortion can be noticed, however we achieved low bit rate and low distortion at the same time. Meanwhile, the analysis over the original and reconstructed versions of several signals, including the ones in Fig. 7, carried out by human experts in disturbance analysis has shown that these distortion levels do not impair their analysis. A detailed description of such tests is beyond the scope of this paper.



(a) Signal s_1



(b) Signal s_2

Fig. 7. Disturbance signals and their reconstructed versions compressed with low bit rate.

One can assess the effectiveness of the proposed method by observing Fig. 8 that depicts the operational curves of the previously shown disturbance signals, which were obtained by using both the reduced and complete set of $\mathbf{b}_k$. The rate is given by bits/sample and the distortion is measured by the MSE (mean-square error). Note that, for both signals, the operational curve obtained by using the reduced set presents a good approximation to the curve obtained by using the complete set. This shows the effectiveness of the training stage, that managed to find a set of dictionaries (specified by $\mathbf{b}_k$) with good performance when applied to electric disturbance signals. This allows us to implement a rate-distortion optimization scheme with low computational complexity, that is also capable of reaching near optimal rate-distortion performance.

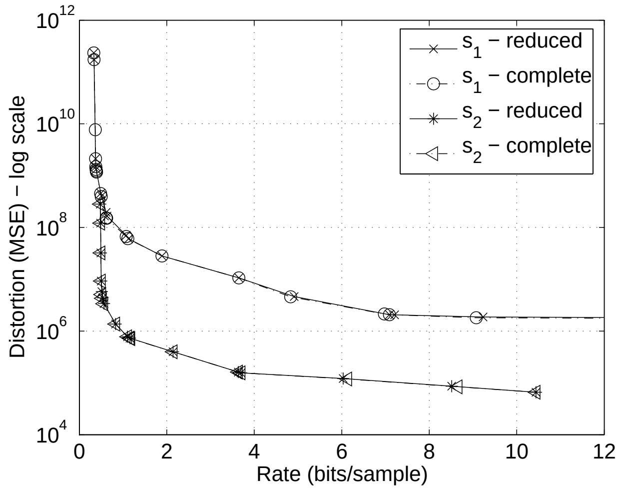


Fig. 8. Operational curves achieved by using the reduced and complete sets of $\mathbf{b}_k$ for signals s_1 and s_2.

V. Conclusions

In this work, a compression framework of atomic decompositions using several dictionaries is proposed. In this case, the R-D optimization corresponds to choose the dictionary that yields the signal approximation having minimum distortion for a desired rate target.

This work addressed the above problem for the particular case of decompositions on parameterized dictionaries, where the several dictionaries are defined by the quantizers applied to the atom parameters. Hence, the R-D problem is solved by finding the coefficient quantizer and the quantizers of the parameters of the atoms which lead to the best rate-distortion compromise.

We obtained the R-D optimum dictionary and coefficient quantizer through the construction of operational curves. In order to decrease the computational complexity of the R-D optimization, we proposed a strategy, of using a training set to select a reduced and good set of parameter quantizers. We assessed the proposed strategy in the framework of electric power system disturbance signals compression. Simulation results showed that the proposed scheme indeed achieves near-optimum R-D performance with low computational complexity.

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