REDUCED CROSS-DISCRIMINATION FOR DISCRIMINATIVE FILTERS

Alexandre P. Mendonça
Instituto Militar de Engenharia - DE/3
Rio de Janeiro, RJ, Brazil, CEP 22290-270
alexmend@ime.eb.br

Eduardo A. B. da Silva
PEE/COPPE/DEL/POLI/UFRJ, Rio de Janeiro, RJ, Brazil
Cx. Postal 68504, CEP 21945-970
eduardo@lps.ufrj.br

ABSTRACT

Image template detection is usually a very important halfway step for a computational vision algorithm. A recently proposed approach for it involves the use of discriminative filters. For a given template, they maximize the energy concentration in a single sample of its output. In this paper, we propose an extension for them that allows multiple template discrimination with reduced cross discrimination, using impulse restoration techniques. Simulation results suggest that the proposed method performs well, being an effective tool for decision problems.
Keywords: *Discriminative Filters, Template Matching, Pattern Recognition, Cross-Discrimination, Impulse Restoration.*

1. INTRODUCTION

The template matching problem has been investigated using several different approaches. Ben-Arie et ali. have been working on a type of template detectors referred to as EXM (***EX****pansion* ***M****atching*) [1-3]. The formulation used there assumes an one-dimensional template that requires either a scanning or some form of rank reduction on the data in the case of multidimensional signals.

In [4-6], there has been proposed a two-dimensional generalization of the methods in [1-3], based on an impulse restoration approach [7-8]. In this paper, we propose an extension of it that allows multiple template discrimination. In addition, we propose a method to avoid incorrect detections that tend to occur when we have similar templates. We refer to this as cross-discrimination reduction.

In section 2, we review the impulse restoration approach for discriminative filter design. In sections 3 and 4, we deal with multiple template discrimination, introducing the discriminative filters of the types OR and WHICH. In section 5, we propose a method for cross-discrimination reduction. Section 6 presents the conclusions.

2. TWO-DIMENSIONAL DISCRIMINATIVE FILTERS

Discriminative filters maximize the energy of an output sample whenever a matching template is found. The *two-dimensional discriminative signal to noise ratio* ($DSNR_2$), defined in [4], is a measure that accounts not only for the maximum energy of a sample, but also considers its energy in relation to the other samples. Thus, for two-dimensional discriminative filters, we need to maximize:

$$DSNR_2 = \frac{c_{i,j}^2}{\left(\sum_m \sum_n c_{m,n}^2\right) - c_{i,j}^2} \quad , \qquad (1)$$

where the coefficients $c_{m,n}$ are obtained after a two-dimensional convolution between an input image window $\mathbf{U} = \{u_{m,n}\}$ and a linear filter $\mathbf{\Theta}$ having impulse response $\{\theta_{m,n}\}$. Note that $\mathbf{\Theta}$ is computed for each different template to be matched. The coefficient $c_{i,j}$ is the one where we wish to concentrate the output signal energy.

One can give an alternative interpretation of discriminative filtering for template detection as follows: the input image $g(m,n)$ can be expressed as

$$g(m,n) = f(m-m_0,n-n_0) + b(m,n) , \qquad (2)$$

where $f(m-m_0,n-n_0)$ is the template centered at position (m_0,n_0) and $b(m,n)$ is the rest of the image.

In order to formulate the impulse restoration problem in matrix notation, an image $g(m,n)$ is transformed in a column vector $\mathcal{G}(k)$ by concatenating its transposed rows. The equivalent template in the unknown position (m_0,n_0) becomes $\mathcal{F}(k-k_0)$ and the noise $b(m,n)$, corresponding to the rest of the image, becomes $\mathcal{B}(k)$. Therefore, equation 2 may be rewritten as

$$\mathcal{G}(k) = \mathcal{F}(k-k_0) + \mathcal{B}(k) = \mathcal{F}(k) * \boldsymbol{\delta}(k-k_0) + \mathcal{B}(k). \qquad (3)$$

or, in a matrix notation,

$$\mathbf{g} = \mathbf{F}\,\boldsymbol{\delta} + \mathbf{b}, \qquad (4)$$

where

$$\mathbf{F} = \begin{bmatrix} \mathcal{F}(0) & \mathcal{F}((2T+1)^2-1) & \cdots & \mathcal{F}(1) \\ \mathcal{F}(1) & \mathcal{F}(0) & \cdots & \mathcal{F}(2) \\ \cdots & \cdots & \cdots & \cdots \\ \mathcal{F}((2T+1)^2-1) & \mathcal{F}((2T+1)^2-2) & \cdots & \mathcal{F}(0) \end{bmatrix} . \qquad (5)$$

It is shown that, using the orthogonality principle, the best estimate of the impulse is [5-8]

$$\hat{\boldsymbol{\delta}} = \mathbf{F}^t (\mathbf{F}\,\mathbf{F}^t + \mathbf{C_b})^{-1} \boldsymbol{g} \quad , \qquad (6)$$

where $\mathbf{C_b}$ is proportional to the noise autocorrelation matrix.

3. DISCRIMINATIVE FILTER OF THE TYPE "OR"

In what follows, we propose a discriminative filter that can discriminate more than one template. For example, we show how to design a filter that can discriminate the letters "A", "E" and "O" from the other letters. We call such filter a *Discriminative Filter of The Type "OR"*.

From [5-8], the best linear estimate $\hat{\boldsymbol{\delta}} = \mathbf{A}\boldsymbol{g}$ of $\boldsymbol{\delta}$ must satisfy the orthogonality principle, that is:

$$E\{(\boldsymbol{\delta} - \hat{\boldsymbol{\delta}})\,\mathbf{g}^t\} = E\{(\boldsymbol{\delta} - \mathbf{A}\,\mathbf{g})\,\mathbf{g}^t\} = \mathbf{0} \quad . \qquad (7)$$

Supposing that the template i corresponds to matrix $\mathbf{F_i}$ (see eq. 5) and has priori probability p_i, equation 7 becomes:

$$\sum_i \; p_i \; E\{(\boldsymbol{\delta} - \mathbf{A}\,\mathbf{F_i}\,\boldsymbol{\delta} - \mathbf{A}\,\mathbf{b})\,(\mathbf{F_i}\,\boldsymbol{\delta} + \mathbf{b})^t\} = \mathbf{0}\,. \quad (8)$$

Supposing that $\boldsymbol{\delta}$ and $\mathbf{b}$ are uncorrelated, it can be shown that,

$$\mathbf{A} = \{\sum_i \; p_i \; \mathbf{F_i}^t\}\,\{\,\mathbf{C_b} + \sum_i \; p_i \; \mathbf{F_i}\,\mathbf{F_i}^t\,\}^{-1}. \qquad (9)$$

To exemplify the use of equation 9, let us consider the case where we want to discriminate the letters "A", "E" or "O" (*Arial*, 8 pt. at a 9x9 window) from the other letters (see fig. 1). Assuming that all probabilities p_i are equal, table 1 shows the discriminative signal to noise ratios of the filter "OR" when the filter $\boldsymbol{\Theta}$ (see in [5-6] how to obtain $\boldsymbol{\Theta}$ from **A**) is applied to the three desired templates "A", "E" or "O" as well as to the other letters. We can see from the $DSNR_2$ values that the type OR filter was able to correctly discriminate the letters "A", "E" or "O" from the others.

Table 1. Performance of OR filter.

	"A"	"E"	"O"	"I"	"U"
$DSNR_2\,(\boldsymbol{\Theta}_{OR})$	0.3221	0.4153	0.7451	0.0001	0.0097

Figure 1. Detected "A","E" or "O" using the OR filter.

4. DISCRIMINATIVE FILTER OF THE TYPE "WHICH"

In the previous section, we presented an algorithm for the computation of the OR filter, that is, a filter which is capable of discriminating any template belonging to a given set. Note that the *discriminative filter of the type "OR"* can only decide if the image contains a template from the set. However, it can not determine which of the templates of the set is contained in the input image. In this section, we propose a change in the computation of the OR filter, in such a way that it can discriminate which of the templates belonging to the set is input to it. We refer to this proposed filter as *discriminative filter of the type "WHICH"*.

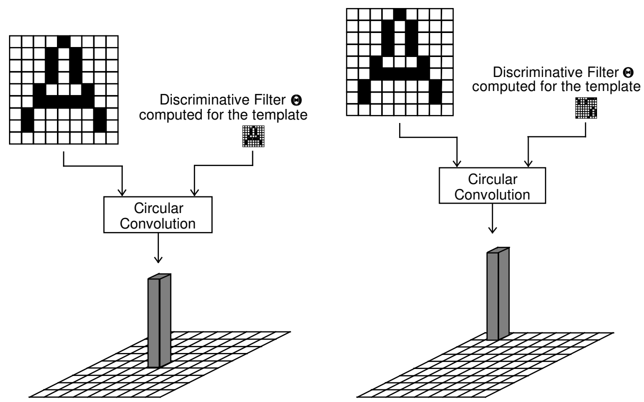
Consider the diagram at the left of figure 2. There, we note the fundamental principle of discriminative filtering: the filter $\boldsymbol{\Theta}$, when circularly convolved with the template "A", offers an output signal with large energy at the center sample.

Consider now the diagram at the right of figure 2. It shows a discriminative filter in which the high-energy output sample is shifted from the center. Note that the discriminative filter was computed for the template "A" circularly shifted right and down.

As it is possible to shift the large energy sample from the output signal, there is an important alternative to compute the discriminative filter of the type OR: we can choose different shifts for each of the templates. Thus, depending on the highest energy output sample location, it becomes possible to decide which of the templates of the set was input. We refer to such filter as discriminative filter of the type WHICH.

When more sophisticated systems are considered, taking a large set of templates, it is possible that some templates are morphologically similar, as the digits "3" and "8" in a 10 digit recognition system. Thus, it is possible that the WHICH filter may not discriminate correctly between them.

In the next section, we propose a method to alleviate this problem. In order to do so, we define the concept of cross-discrimination, that is, how much one template can be wrongly discriminated as being another, and then propose an algorithm that reduces the effects of the cross-discrimination in discriminative filters of the type WHICH.



(a) (b)

Figure 2. (a) Centered impulse. (b) Shifted impulse.

5. CROSS DISCRIMINATION REDUCTION

We start by analyzing the performance of a WHICH filter designed to recognize the 10 numeric digits (*Arial*, 8 pt.). In order to provide a larger sample separation between the impulses corresponding to the different letters, we interpolate each character by 3, generating 27x27 patterns. To objectively measure the cross-discrimination, we define the cross-discriminative SNR as

$$XDSNR_2(\boldsymbol{\alpha},\boldsymbol{\beta}) = \frac{c_{\beta i,\,\beta j}^{\;2}}{\left(\sum_m \sum_n c_{m,n}^{\;2}\right) - c_{\beta i,\,\beta j}^{\;2}}\;, \quad (10)$$

where $\mathbf{C} = \{c_{m,n}\} = \boldsymbol{\alpha} * \boldsymbol{\Theta}_{WHICH}$ and (β_i,β_j) are the sample coordinates associated to the detection of a template $\boldsymbol{\beta}$. This equation gives a measure of how much energy of the output of the WHICH filter for template $\boldsymbol{\alpha}$ is concentrated at the sample corresponding to the detection of a template $\boldsymbol{\beta}$.

Table 2 shows selected $XDSNR_2$ values for the 10 numeric digits example. The underlined entries in the table correspond to high values of cross-discrimination. By observing them, we see that there is possibility of false detections for the patterns "3", "6", "8", "9" or "0". In the ideal case, we should have large entries only at the main diagonal.

In what follows, we describe a process to design a discriminative filter with reduced cross discrimination. We start by designing a discriminative filter that will detect template $\boldsymbol{\alpha}$ and not detect template $\boldsymbol{\beta}$. Detection of template $\boldsymbol{\alpha}$ corresponds to a peak at coordinates (α_i,α_j); detection of template $\boldsymbol{\beta}$ would correspond to a peak at coordinates (β_i,β_j). We shift template $\boldsymbol{\beta}$, generating template $\boldsymbol{\beta}'$ such that its detection would correspond to a peak at coordinates (α_i,α_j). Therefore, in such filter, we want to maximize the amplitude of the estimated impulse at (α_i,α_j) if the template $\boldsymbol{\alpha}$ is input ($\hat{\boldsymbol{\delta}}\,|\boldsymbol{\alpha}$), while minimizing the amplitude of the estimated impulse at (α_i,α_j) if template $\boldsymbol{\beta}'$ is input ($\hat{\boldsymbol{\delta}}\,|\boldsymbol{\beta}'$).

Note that, in the discriminative filter, the random vector $\boldsymbol{\delta}$ (fig. 3-a) is the one who marks the position (α_i,α_j) indicating that template $\boldsymbol{\alpha}$ is present. Since $\hat{\boldsymbol{\delta}}$ is a linear estimator of $\boldsymbol{\delta}$, and $\boldsymbol{\delta}$ has the maximum energy concentration at (α_i,α_j), then ($\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\alpha}$) and ($\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\beta}'$) will be positive at (α_i,α_j) (see fig. 3). Therefore, since we want to maximize $\hat{\boldsymbol{\delta}}\,|\boldsymbol{\alpha}$ and minimize $\hat{\boldsymbol{\delta}}\,|\boldsymbol{\beta}'$ at (α_i,α_j), then it suffices to minimize ($\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\alpha}$) - K ($\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\beta}'$) at (α_i,α_j). K is a constant chosen in order to guarantee that the expression is positive at (α_i,α_j). In the digits example, this restriction gives a maximum value of K equal to 0.95. Also, since the energy of ($\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\alpha}$) - K ($\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\beta}'$) is concentrated at (α_i,α_j), then the above minimization is equivalent to minimization of

$$\zeta\,(\mathbf{F}_{\boldsymbol{\alpha}},\mathbf{F}_{\boldsymbol{\beta}'},\mathbf{A},\mathbf{C_b}) = \mathrm{E}[\|\,(\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\alpha}) - K\,(\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\beta}')\,\|^2] \quad . \quad (11)$$

Since

$$\hat{\boldsymbol{\delta}}\,|\boldsymbol{\alpha} = \mathbf{A}\,\mathbf{g}_{\boldsymbol{\alpha}} = \mathbf{A}\,\mathbf{F}_{\boldsymbol{\alpha}}\,\boldsymbol{\delta} + \mathbf{A}\,\mathbf{b} \quad , \quad (12)$$

$$\hat{\boldsymbol{\delta}}\,|\boldsymbol{\beta}' = \mathbf{A}\,\mathbf{g}_{\boldsymbol{\beta}'} = \mathbf{A}\,\mathbf{F}_{\boldsymbol{\beta}'}\,\boldsymbol{\delta} + \mathbf{A}\,\mathbf{b} \quad , \quad (13)$$

equation 11 is equivalent to

$$\zeta\,(\mathbf{F}_{\boldsymbol{\alpha}},\mathbf{F}_{\boldsymbol{\beta}'},\mathbf{A},\mathbf{C_b}) = (1-K)^2\,\mathrm{E}\,\{\,\|\,\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}_{\boldsymbol{\gamma}}\,\|^2\} \quad , \quad (14)$$

where $\mathbf{F}_{\boldsymbol{\gamma}} = (\mathbf{F}_{\boldsymbol{\alpha}} - K\mathbf{F}_{\boldsymbol{\beta}'})/(1-K)$, $\mathbf{g}_{\boldsymbol{\gamma}} = \mathbf{F}_{\boldsymbol{\gamma}}\,\boldsymbol{\delta} + \mathbf{b}$ and $\hat{\boldsymbol{\delta}}_{\boldsymbol{\gamma}} = \mathbf{A}\,\mathbf{g}_{\boldsymbol{\gamma}}$.

Equation 14 indicates that, for minimizing $\zeta\,(\mathbf{F}_{\boldsymbol{\alpha}},\mathbf{F}_{\boldsymbol{\beta}'},\mathbf{A},\mathbf{C_b})$, it is necessary to minimize $\mathrm{E}\,\{\,\|\,\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}_{\boldsymbol{\gamma}}\|^2\,\}$, that can be considered as the impulse estimation error that the operator $\mathbf{A}$ offers to the template $\mathbf{F}_{\boldsymbol{\gamma}} = (\mathbf{F}_{\boldsymbol{\alpha}} - K\mathbf{F}_{\boldsymbol{\beta}'})/(1-K)$. Consequently, the solution for the minimum of $\zeta\,(\mathbf{F}_{\boldsymbol{\alpha}},\mathbf{F}_{\boldsymbol{\beta}'},\mathbf{A},\mathbf{C_b})$ is equivalent to compute $\mathbf{A}$ such that $\mathbf{A}\,\mathbf{g}_{\boldsymbol{\gamma}}$ offers the minimum mean square error for the impulse that marks the position of $\mathbf{F}_{\gamma}$. Since the observed image is $\mathbf{g}_{\alpha}$, then, using the orthogonality principle, $\zeta\,(\mathbf{F}_{\boldsymbol{\alpha}},\mathbf{F}_{\boldsymbol{\beta}'},\mathbf{A},\mathbf{C_b})$ is minimized when

$$\mathrm{E}\,\{\,\{(\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\alpha}) - K\,(\boldsymbol{\delta}-\hat{\boldsymbol{\delta}}\,|\boldsymbol{\beta}')\,\}\,\mathbf{g}_{\boldsymbol{\alpha}}^{t}\,\} = \mathbf{0} \quad . \quad (15)$$

Table 2. $XDSNR_2$ after convolving the $\boldsymbol{\Theta}_{WHICH}$ and selected templates.

α \ β	"3"	"6"	"8"	"9"	"0"
"3"	**0.1127**	0.0336	*0.0854*	*0.0448*	*0.0627*
"6"	*0.0460*	**0.1281**	*0.0542*	0.0257	*0.0390*
"8"	*0.0623*	*0.0743*	**0.1466**	0.0279	*0.0679*
"9"	0.0149	0.0207	0.0142	**0.0824**	*0.0544*
"0"	*0.0408*	*0.0610*	*0.0426*	0.0258	**0.0877**

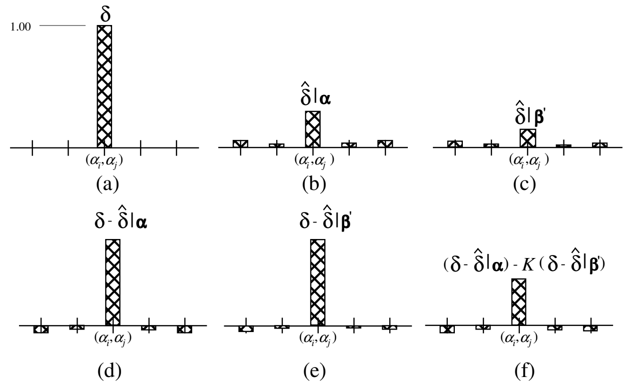


Figure 3. Searching for reduction cross discrimination indicators.

Therefore, using the condition of equation 15, the operator $\mathbf{A}$ (from where we will take $\boldsymbol{\Theta}$, see [5-6]) can be expressed as

$$\mathbf{A} = (1-K)\mathbf{F}_{\boldsymbol{\alpha}}^{t}\,\{\,(\mathbf{F}_{\boldsymbol{\alpha}} - K\,\mathbf{F}_{\boldsymbol{\beta}'})\mathbf{F}_{\boldsymbol{\alpha}}^{t} + (1-K)\mathbf{C_b}\}^{-1} \quad . \quad (16)$$

For the case of multiple discrimination, the left hand side of equation 15 should be replaced by a term considering, for each template $\boldsymbol{\alpha}_m$, all possible templates $\boldsymbol{\beta}_n$. Note that each template $\boldsymbol{\alpha}_m$ should be weighted by its a priori probability p_m (see eq. 9). Since, from the discussion above, K in equation 15 weights the influence of template $\boldsymbol{\beta}'$ in the detection of $\boldsymbol{\alpha}$, we define $K_{m,n}$ as a constant that weighs the influence of template m in template n. We also define $\mathbf{F}_{m,n}$ as the matrix $\mathbf{F}$ of template n shifted so that the energy peak resulted after convolution is shifted to the same sample of the peak resulted after convolution between the WHICH filter and m. Using this notation, equation 16, for the multiple discrimination case, would be

$$\mathbf{A} = \{\sum_m\sum_n\; p_m\,(1-K_{m,n})\,\mathbf{F}_{m,m}^{t}\}\,\{(1-\sum_m\sum_n\; p_m\,K_{m,n})\,\mathbf{C_b} + \sum_m\sum_n\;\{p_m\,(\mathbf{F}_{m,m} - K_{m,n}\,\mathbf{F}_{m,n})\mathbf{F}_{m,m}^{t}\}\}^{-1} . \quad (17)$$

To evaluate the proposed method, we use the 10 numeric digits example. Note that the matrix $\mathbf{K} = \{\,K_{m,n}\,\}$ should be experimentally adjusted. The initial guess is provided by inspecting table 2; the higher the values of $XDSNR_2$ off the main diagonal, the higher the values of the corresponding $K_{m,n}$ should be. It should be noted that it is necessary to adjust $\mathbf{K}$ step by step until the $XDSNR_2$ values are within the desired range. For the 10 digits example, the best $\mathbf{K}$ found is

$$\mathbf{K} = 0.1\begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -5 & 0 & 0 & 5 & 0 & 6 & 0 & 1 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 5 & 0 & -4 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 10 & 3 & -4 \end{bmatrix} . \quad (18)$$

The corresponding $XDSNR_2$ values for selected templates are shown in table 3.

In order to objectively evaluate the performance of the proposed method, we define the discriminative efficiency $D_E(\boldsymbol{\alpha})$ as

$$D_E(\alpha) = \frac{XDSNR_2(\alpha,\alpha)}{\max_\beta \{ XDSNR_2(\alpha,\beta) \}} , \qquad (19)$$

Using equation 19, the best matrix **K** is the one which maximizes $\min_\alpha\{ D_E(\alpha) \}$. Figure 4 shows graphically the values of $D_E(\alpha)$ for each template obtained with both the original and the modified WHICH filter for the digits example (eqs. 17-18). The $\min_\alpha\{ D_E(\alpha) \}$ increased from 1.3197 (original WHICH filter) to 1.7080 (eqs. 17-18), that is, nearly 30 %. Note that, using **K**, the smaller values (templates “3”, “6”, “8”, “9”and “0”) are more similar than as in the original WHICH filter.

To simulate the proposed method for cross discrimination reduction, a set with 10 images was synthesized, where each image has 20 occurrences of a specific digit ("0" to "9") corrupted by uniform additive noise. The SNR is 3 dB. Results for the modified as well as the original WHICH filter are shown in table 4.

As expected, the modified WHICH filter performs better than the original WHICH filter. The hit ratio increased from 86 % to 91 %.

Table 3. $XDSNR_2$ after convolving the $\Theta_{K\text{-}WHICH}$ with selected templates.

α \ β	"3"	"6"	"8"	"9"	"0"
"3"	**0.1201**	0.0237	*0.0697*	*0.0532*	*0.0605*
"6"	*0.0522*	**0.0923**	*0.0435*	0.0306	*0.0388*
"8"	*0.0692*	*0.0556*	**0.1199**	0.0326	*0.0702*
"9"	0.0175	0.0160	0.0130	**0.1071**	*0.0537*
"0"	*0.0456*	*0.0461*	*0.0351*	0.0354	**0.0853**

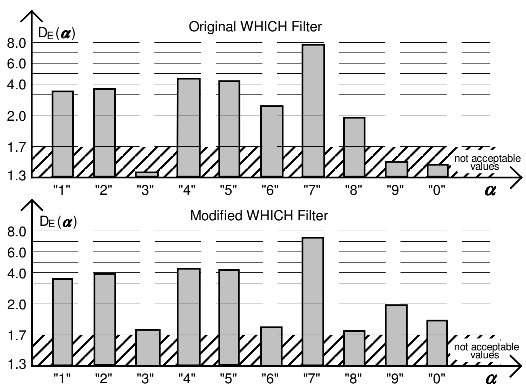


Figure 4. Performance comparison using $D_E(\alpha)$.

Table 4. Performance using both original and modified WHICH filter.

Original WHICH Filter	times	%
correct detections	172 / 200	86
changed detections	27 / 200	13.5
missed detections	1 / 200	0.5
false detections	-	-
Modified WHICH Filter	times	%
correct detections	182 / 200	91
changed detections	17 / 200	8.5
missed detections	1 / 200	0.5
false detections	-	-

6. CONCLUSIONS

In this paper, we first reviewed the two-dimensional discriminative filters. Their main objective is to obtain a two-dimensional filter that, when convolved with the image template, generates as output an image with the energy concentrated in only one sample.

We also presented a method based on an extension of the impulse restoration problem for the multiple template detection case. Using it, we proposed a closed-form solution for two types of discriminative filters: the OR filter and the WHICH filter. While the OR filter can only decide if the input image belongs to a template set, the WHICH filter can tell which of the templates is the input image.

Then, we proposed a discriminative filter design method with reduced cross discrimination. It was shown that such filters can provide significant improvements over the original WHICH filter in pattern recognition systems.

The results shown are promising, and indicate that discriminative filters can be used as viable alternatives for complex non linear classifiers.

A suggestion for further developments is the investigation of methods for incorporating rotation and scaling invariance in the design method, as in [9].

7. REFERENCES

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