COHERENT DECOMPOSITIONS OF POWER SYSTEMS SIGNALS USING DAMPED SINUSOIDS WITH APPLICATIONS TO DENOISING

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ABSTRACT

In this paper we propose a method to decompose electric power systems signals using damped sinusoids. It is inspired in the matching pursuits algorithm. The matching pursuits algorithm has the potential of decomposing a signal into coherent components. However, it is not a trivial task to obtain a decomposition which is meaningful in physical terms, as it is desirable in the case of power systems signals. We describe enhancements to the basic matching pursuits algorithm that generate decompositions in damped sinusoids with high correlation to the actual physical phenomena. We test the effectiveness of the proposed method in a signal denoising application, where the physically meaningful components are extracted from signals corrupted by noise. From the extracted components, a version of the signals almost free from noise are obtained.

1. INTRODUCTION

In power systems plants it is necessary to monitor the quality of the power delivered as well as to detect and identify failures in the system. The most common approach is to sample the signals corresponding to the power delivered (the current and/or the voltage) and store or transmit them for future analysis by a system expert.

A problem that arises is how to compact/represent the power systems signals with high quality at good compression ratios. A good model for power system signals based on damped sinusoids was proposed in [1]. At this model the signals are represented by a sum of damped sinusoids (in fact damped harmonics of a fundamental frequency F):

$$x(t)=\sum_{q=0}^{Q-1} A_q\cos(2\pi k_qFt+\phi_q)e^{-\lambda_q(t-t_{0_q})} \times[u(t-t_{0_q})-u(t-t_{f_q})], \tag{1}$$

Each component is represented by a 6-tuple (A_q, k_qF, λ_q, ϕ_q, t_{0_q}, t_{f_q}), where A_q represents the amplitude, k_qF the frequency, λ_q the damping factor, ϕ_q is the phase, and t_{0_q} and t_{f_q} represent the starting and ending times of a component (being $u(t)$ the unit step function). Note that the frequencies of the sinusoids are integer multiples of the fundamental frequency F. By representing the signal by a set of these 6-tuples, one would obtain a very compact and accurate representation of the signal.

This paper is organized as follows: in section 2 we present the method used to obtain decompositions as in eqn. (1). Since such decompositions tend to have an infinite number of terms, in section 3 we deal with the problem of finding ways to automatically stop such decompositions while assuring that the phenomena present in the signal are preserved. In section 4 we assess the effectiveness of the proposed method by analysing its performance in denoising applications.

2. ADAPTIVE DECOMPOSITIONS OF SIGNALS

A signal $x(t)$ can be represented as a sum of other predefined signals $f_k(t)$, $k\in\{0,1,\ldots,K\}$ as [2]:

$$x(t)=\sum_{n=0}^{M-1}\alpha_n f_{k(n)}(t). \tag{2}$$

We refer to $f_k(t)$ as atoms or structures. The set of all possible atoms $f_k(t)$ is the so called dictionary $\mathbb{D}$. In eqn. (2) M signals given by $f_{k(n)}(t)$ are used to represent $x(t)$. α_n is an energy factor for the signal $f_{k(n)}(t)$. An important point related to the representation in eqn. (2) is that it decomposes the signal $x(t)$ as a linear combination of M other signals. Therefore, it has the potential of, depending on the choice of the dictionary $\mathbb{D}$, allowing the identification of different phenomena (signals) composing $x(t)$, yielding a compact representation [3]. The decomposition of eqn. (2) is equivalent to that presented at eqn. (1), where the functions that compose the dictionary are explicitly shown. The key point in such decompositions is how to obtain the coefficients α_n and the atoms $f_{k(n)}(t)$. We accomplish this using the **MP** algorithm.

2.1. Matching Pursuits

The **MP** algorithm was first introduced by *Mallat* and *Zhang* [4]. The **MP** algorithm is iterative, and at each step chooses the function from the dictionary that has the higher inner product with the signal. The signal is therefore represented by successive approximations.

Suppose we want to decompose the signal $\mathbf{x}$. Define a dictionary $\mathbb{D}=\{\mathbf{g}_\gamma\}$, $\gamma\in\Gamma$, (γ is a set of parameters and

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Γ is the set of all possible γ), such that $\| \mathbf{g}_\gamma \| = 1$. If $\mathbb{D}$ is complete [2, 4], we can represent $\mathbf{x}$ as a sum of elements of the dictionary $\mathbf{g}_{\gamma(n)}$, that we call atoms or structures:

$$\mathbf{x} = \sum_n \alpha_n \mathbf{g}_{\gamma(n)}. \tag{3}$$

In order to compute the coefficients α_n we can choose $\mathbf{g}_{\gamma(0)}$ such that $\alpha_0 = \langle \mathbf{x}, \mathbf{g}_{\gamma(0)} \rangle = \max_{\gamma \in \Gamma} \langle \mathbf{x}, \mathbf{g}_\gamma \rangle$, and then split $\mathbf{x}$ in two parts defining $\mathbf{R}^0{}_\mathbf{x} = \mathbf{x} - \alpha_0 \mathbf{g}_{\gamma(0)}$. Carrying out this process iteractively, we can compute the $n+1$ order residue, $\mathbf{R}^{n+1}{}_\mathbf{x}$, as:

$$\mathbf{R}^{n+1}{}_\mathbf{x} = \mathbf{R}^n{}_\mathbf{x} - \alpha_{n+1} \mathbf{g}_{\gamma_{n+1}}, \tag{4}$$

where $\alpha_{n+1} = \langle \mathbf{R}^n{}_\mathbf{x}, \mathbf{g}_{\gamma_{n+1}} \rangle$ which is obtained with the γ, that gives its maximum value. It is computed at step $n+1$ from the residue of step n. Observe that $\mathbf{R}^n{}_\mathbf{x}$ and $\mathbf{g}_{\gamma(n)}$ are orthogonal and therefore the signal energy is conserved [4]. That is, at a given step m of the decomposition:

$$\| \mathbf{x} \|^2 = \sum_{n=0}^{m-1} \| \langle \mathbf{R}^n{}_\mathbf{x}, \mathbf{g}_{\gamma(n)} \rangle \|^2 + \| \mathbf{R}^m{}_\mathbf{x} \|^2. \tag{5}$$

and therefore the energy of the residual decreases in each approximation pass.

2.2. Damped Sinusoids Dictionary

The **MP** was proposed using the *Gabor* dictionary [4, 2]. These atoms are defined by:

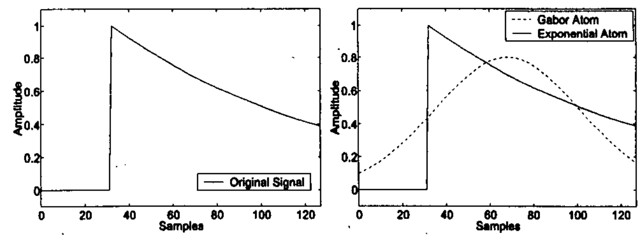
$$g_\gamma(t) = \frac{1}{\sqrt{s}} 2^{\frac{1}{4}} e^{-\pi\left(\frac{t-u}{s}\right)^2} \cos(\xi t + \phi) \tag{6}$$

where $\gamma = [s, u, \xi, \phi]$ – (s is the scale, u is the center of the atom, ξ is the atom frequency and ϕ is the phase). The aspects of the implementation of the *Matching Pursuits* with the *Gabor* dictionary are discussed in [4, 5]. An important feature of this dictionary is that the parameter space of the atoms can be sampled [4, 5] while still obtaining a complete dictionary [2, 4]. A number of other different dictionaries has been proposed, namely: trained, stochasticaly generated or even a collection of dictionaries (library). Here we want to use a dictionary based on the electric signals model of eqn. (1), yielding atoms defined by:

$$f_\gamma(t) = \cos(\xi t + \phi) e^{-\lambda(t-t_0)} \times [u(t-t_0) - u(t-t_f)]. \tag{7}$$

where $\gamma = [\lambda, \xi, \phi, t_0, t_f]$.

The problem with this dictionary is that the sampling of the parameter set $\gamma = [\lambda, \xi, \phi, t_0, t_f]$ for the atoms of eqn. (7) is not trivial. Here we propose to solve this problem by obtaining the decomposition into a dictionary of functions f_γ (see eqn. (7)) indirectly from the decomposition into the dictionary composed by g_γ (see eqn. (6)). In the following subsection we describe the general guidelines of the algorithm. It is important to note that in the case of the functions presented so far, that is, the *Gabor* atoms and the damped sinusoids, there is no need to sample the atom's phase in order to construct the dictionary. It can be computed using a complex representation of them. Details can be found in [3, 5].



(a) *Original* (b) *Decomposed*

Fig. 1. Exponential Signal.

2.3. The Decomposition Algorithm

First the *Gabor* atom, eqn. (6), that best matches the signal is obtained with the **MP** algorithm. Once this atom is chosen, we use it as a guess to an exponential atom, eqn. (7), that best matches the signal. This is done as follows:

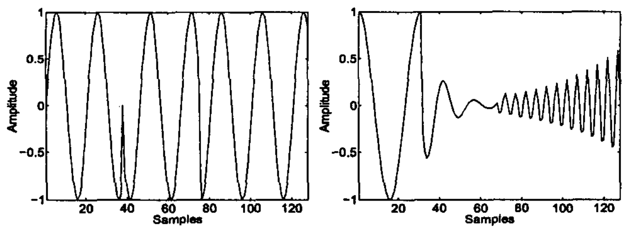
1. The *Gabor* atom that best matches the signal is found by the **MP** at the step.
2. With the parameters of the *Gabor* atom as a initial guess, $\gamma = [s, u, \xi]$, find the exponential atom that best matches g_γ in one of the half-planes defined by the simmetry axis of the *Gaussian* envelope. This is done using a pseudo-*Newton* algorithm.
3. Having the parameters of the damped sinusoid atom, $\gamma = [\lambda, \xi, t_0]$, compute the optimum phase for it. After that, the best time support of the atom (t_0 and t_f) should be searched by maximizing the inner product between the atom and the signal and by minimizing the error in the support region. From these values we perform another pseudo-*Newton* search for the parameters $[\lambda, \xi]$, obtaining the 5-tuple that characterizes the atom given by $\gamma = (\xi, \lambda, \phi, t_0, t_f)$. The norm of the atom is given by $A = \langle R^n{}_\mathbf{x}, g_\gamma \rangle$.

In fig. 1 we show an exponential signal decomposed with the **MP** using the *Gabor* dictionary as in [2, 4] and the damped sinusoid dictionary using the above algorithm. We observe that the proposed search for the damped sinusoid based on the *Gabor* atom gives a good reconstructed signal, with an SNR of 210.68 dB.

However, for power systems signals the atoms thus found often do not correspond to physically meaninful phenomena. For example:

I. Signals formed by the concatenation of two or more sinusoids of same frequency but different amplitudes can be confused with a single damped sinusoid. To avoid this problem, after finding a damped sinusoid, we search for a pure sinusoid having the same frequency as the chosen atom. Its best time support is also searched. If the inner product between the pure sinusoid in its support region and the signal is larger then $\lambda_{opt} A$ ($\lambda_{opt} \leq 1$ is an optimality factor) and the residue per sample in the support region is smaller for the pure sinusoid then for the damped sinusoid, we replace the damped sinusoid by the pure sinusoid found. A full description of this process is can be found in [6].

II. When there are concatenated sinusoids of same frequen-



(a) *S1* (b) *S2*

Fig. 2. Synthetic Signals

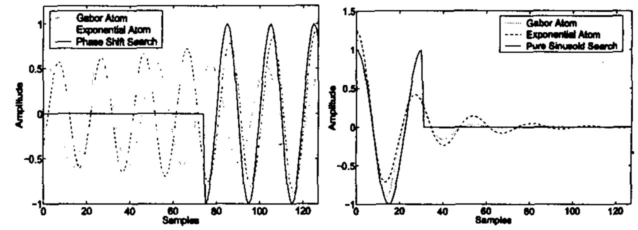
cy and amplitude but different phase (see fig. 2(a)) , the sinusoids tend to be grouped as one damped sinusoid atom with phase that maximizes the inner product with the signal. We avoid that by generating a pure sinusoid having the same frequency as the chosen damped sinusoid, searching for its best time support as in **1** above. By comparing the inner product and the error in the support region of both atoms this type of error can be avoided. A full description of this process is described in [6].

Using these procedures the atoms found are physically meaninful. This can be confirmed by the decomposition of the two synthetic signals *S1* and *S2* (see fig. 2) that were generated with the model of eqn. (1). The first steps of their decomposition are depicted in fig. 3. Note that signal *S1* corresponds to case **II** above and signal *S2* corresponds to case **I** above. In these examples we see the effectiveness of the *Gaussian* to exponential envelope matching in both cases, as well as the pure sinusoid detection procedure for the case of signal *S2* and the phase shift detection algorithm for the case of signal *S1*.

3. COHERENT DECOMPOSITIONS

Important features of the decomposition presented so far are: **1**. The representation is adaptive since at each step the most similar function to the signal is chosen to approximate the signal from the dictionary $\mathbb{D}$; **2**. At step n we have an approximation of the signal [2] with the dictionary structures selected up to this step. **3**. It is capable of correctly identifying structures in the power systems signals corresponding to physically meaningful phenomena. However, after a sufficiently large number of steps, all the physically meaningful structures will have been represented; this implies that the structures found after that will just correspond to noise. Therefore, a question that remains is at which step n to stop the decomposition process. In other words, one should be capable of identifying at which step we start to represent just noise. One possible solution to this problem could be to halt the process given a prescribed error criterion. This solution is not optimal as it does not take into consideration that: **A**. The structures composing the signal can have in principle any amplitude level, making it difficult to set up a fixed threshold; **B**. The signal can be noisy.

We solve this problem by measuring how much of the signal is being approximated at step n. This can be done through the approximation ration [4] defined below:

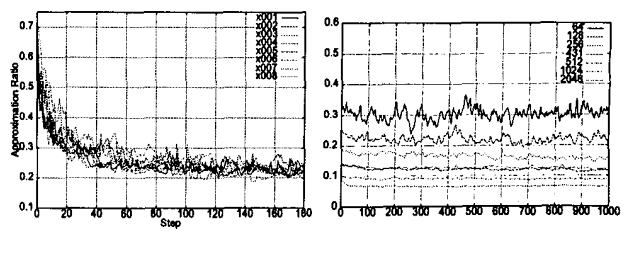


(a) *S1* (b) *S2*

Fig. 3. First step decomposition of the synthetic Signals

$$\lambda(\mathbf{R}^n{}_\mathbf{x}) = \frac{\| \langle \mathbf{R}^n{}_\mathbf{x}, \mathbf{g}_{\gamma(n)} \rangle \|}{\| \mathbf{R}^n{}_\mathbf{x} \|}. \qquad (8)$$

Davis [7] defines the concept of coherent structures of the decomposition as being the ones with higher approximation ratio at the decomposition process. The power of this concept comes from the fact that in [4] it is described that after a large number of iterations of the **MP** the approximation ratio stabilizes around a given value that is independent of the signal being decomposed; in addition, for *Gaussian* noise the approximation ration stabilizes after only a few iterations. This implies that after the approximation ratio stabilizes, then it means that we are only approximating noise, and the decomposition can stop. Then, in practical terms, a decomposition should stop when its approximation ratio reaches a threshold. This threshold depends only on the dictionary, and can be determined by decomposing *Gaussian* noise. In figure 4(a) one can see the approximation ratios for different signals of length 128 decomposed using the **MP** with a *Gabor* dictionary. One can see that for large n it, in fact, oscilates around an average value. Because of this, in practice, we compute an average of the approximation ratio over a window ($\lambda_{\text{avg}_f}(\mathbf{R}^M{}_\mathbf{x})$), in order to filter out these small oscillations. The average approximation ratio is plotted in fig. 4(b) for *Gaussian* noise signals of different lengths. It is important to note that the value to which the average approximation ratio converges ($\lambda_{\text{avg}_f}(\mathbf{R_g})$) varies with the dictionary and decreases with the signal dimension. Table 1 shows the experimental values of the average approximation ratios for the damped sinusoids dictionary for several signal lengths.



(a) $\lambda(\mathbf{R}^n{}_\mathbf{x})$ for different signals of length 128.

(b) $\lambda_{\text{avg}_f}(\mathbf{R}^M{}_\mathbf{x})$ ratio for *Gaussian* noise signals.

Fig. 4. Approximation ratio behavior with the *Gabor* dictionary.

Table. 1. Average approximation ratio for different size noise signal in the **MP** with the damped sinusoids dictionary.

Signal Size	64	128	256	512	1024
$\lambda_{\text{avg}_f}(\mathbf{R_g})$	0.39	0.28	0.22	0.14	0.087

Therefore, if we carry out the decomposition only until $\lambda_{\text{avg}_f}(\mathbf{R}^M\mathbf{x}) \geq \lambda_{\text{avg}_f}(\mathbf{R_g}) + \varepsilon$, we have that only the coherent structures (the ones with approximation ratios higher than the ones of the noise signal) are selected. Thus, this is a good stopping criterion for the decomposition process. Note that, since only the structures that are more coherent than the noise signal are kept, this process can also be used for signal denoising (see [2]).

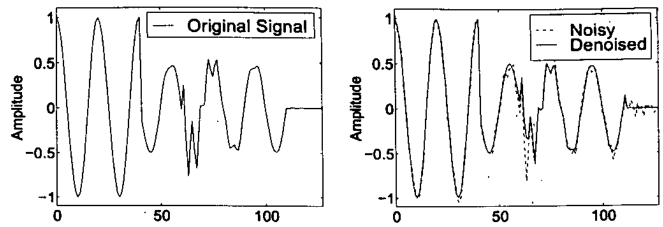
4. RESULTS

In order to assess the effectiveness of the proposed method to stop the decomposition process, yielding a coherent representation, we studied the performance of the proposed algorithm when decomposing a signal with different levels of noise added. The energy of the noise added can be seen in table 2. We also present the number of structures into which the signal was decomposed together with the SNR of the reconstructed signal. The signal used for this study was generated according to the model of eqn. (1) with 4 structures and is shown at fig. 5(a). Table 2 shows the number of structures identified by the algorithm. The algorithm identifies successfully the structures up to a noise level of about 5% of the signal energy. The signal in the last line of the table is shown in fig. 5(b).

From these results we can observe that: **1.** Small energy noise influences the number of structures used in the algorithm; however, it does not influence which atoms are chosen; therefore, noise can be added to a signal in order to control the number of structures used in its representation, that is, the more noise we add the earlier the step we start approximating noise; **2.** the addition of a small amount of noise maybe handy to help us to find suitable decompositions in some problematic cases; **3.** the algorithm is capable of sensing the presence of noise in the signal, this idea can be used to perform denoising by synthesis [2]. Thus, if we have an electric signal corrupted by the addition of noise, we can use the proposed algorithm with the damped sinusoids dictionary to represent the noisy signal and reconstruct it "denoised" from the adaptive decomposition.

Table 2. Decomposition of a signal, using a damped sinusoids dictionary, with addition of different intensities of noise.

Noise Energy	$\frac{\text{Signal Energy}}{\text{Noise Energy}}$	Structures	SNR dB
0.0238	1,277.4	14	77.95
0.0809	375.81	5	46.31
0.3237	93.92	5	45.36



(a) Original signal (b)

Fig. 5. (a) Original signal; (b) Signal with added noise of energy 0.3237 (dashed line) together with the reconstructed signal (continuous line).

5. CONCLUSIONS

In this article we propose an algorithm for the representation of power systems signals using damped sinusoids. The proposed method automatically identifies physically meaningful signal structures, and therefore is able to differentiate signal from noise. Experiments with noisy signals have shown that the proposed method performs very well as a signal denoising tool.

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