Design of High Performance Wavelets for Image Coding

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Abstract

This paper addresses the design of high performance wavelets for image coding using a perceptual criterium defined as the product of the theoretical coding gain and an index recently reported, called peak-to-peak ratio (PPR). It presents some new results in biorthogonal linear-phase wavelet design for image compression. A simplified design procedure was developed using a technique to generate high-order filters from lower-order ones. With this function the perceptual quality expected for the compressed images can be evaluated without having to go through tests with experts. Although this technique is sub-optimal, it yields wavelets which are optimal or very close to it. Some examples are given.

1 Introduction

Design of wavelets with optimum characteristics for image compression is an important issue, still attracting the interest of image processing community. In [1] it was verified that many of the performance indexes used in wavelet design are of questionable interest when used for images. Features such as frequency selectivity, regularity and even coding gain show little correlation with the subjective evaluation of coded images. Instead, an index based on time response of wavelets, called Peak-to-Peak ratio (PPR), was found to be important in the subjective evaluation. The PPR measures the damping of the oscillations of the synthesis wavelet towards its tails. Its calculation can be found in [1], [3]. Such oscillations have direct influence on the visualization of quantization errors after image reconstruction.

Optimum design based on perceptual criteria is troublesome because it usually involves subjective tests which cannot be put into an optimization loop in an efficient way. Since the product of the PPR and the coding gain (G_c) shows high correlation to perceptual quality of coded images [1], it can be used to circumvent this limitation.

In [3] it was presented a design technique for biorthogonal linear-phase wavelets aimed at obtaining high values of $G_c \times PPR$. The technique consists of incrementally increasing the order of a starting wavelet through a series of one dimension optimizations. Despite the restricted search space, good results were attained.

This work has two main contributions: first it extends the technique presented in [3] by performing multidimensional optimizations; then an analysis of the resulting wavelets is performed highlighting the trade-offs among $G_c \times PPR$, length of the prototype filters, ringing and blocking artifacts. It is concluded that very good wavelets can be obtained while keeping their orders relatively low.

In this work, the model of the input image signal is assumed to be an auto-regressive process of order 1 (AR(1)), with autocorrelation $\rho = 0.95$. We use the theoretical coding gain G_c from Katto and Yasuda [4].

2 Filter Design

The construction of a wavelet decomposition scheme requires, as a prototype, a suitable lowpass/highpass filter pair, or equivalently a two-band filter bank (FB). For image processing biorthogonal filter banks with linear phase filters are preferred.

Biorthogonality in a two-band FB is equivalent to

$$H_0(z)\,H_1(-z) - H_0(-z)\,H_1(z) = 2z^{-2m+1} \qquad (1)$$

where $H_0(z)$ is the analysis lowpass filter of length L_0 and $H_1(z)$ is the analysis highpass filter of length L_1.

There is only one filter $H_1(z)$ of order less than or equal to the order of $H_0(z)$ ($L_1 \leq L_0$) satisfying eq. (1), see [3] and [5]. The synthesis lowpass filter $G_0(z)$, of length L_1, and the highpass filter $G_1(z)$, of length L_0, can be determined from the pair $(H_0(z), H_1(z))$. So, one can optimize the wavelet filter bank using any design criterium by simply changing the coefficients of $H_0(z)$. When it is desired that $L_1 \geq L_0$ the optimization can be performed by using $H_1(z)$ or $G_0(z)$ instead of $H_0(z)$.

3 Wavelet optimization

Wavelet improvement is obtained by the combination of two techniques: increment in prototype filter order and multivariable coefficient optimization:

- Given a wavelet generated by the pair $(H_0(z), H_1(z))$ we build a new pair $(H_0^{(1)}(z), H_1^{(1)}(z))$ by increasing the prototype filter order as follows:

$$H_0^{(1)}(z) = 1 + kz^{-1}H_0(z) + z^{-(L_0+1)} \qquad (2)$$

  where k is a factor to be optimized. $H_1^{(1)}(z)$ is found from eq. (1). If $L_1 > L_0$, $G_0^{(1)}(z)$ is used instead.

- Given a wavelet generated by the linear-phase pair $(H_0(z), H_1(z))$ we just optimize the coefficients of $H_0(z)$ (or $G_0(z)$ if $L_1 > L_0$). It should be noted that a linear-phase filter of length L has only $trunc((L-1)/2)$ degrees of freedom because the first coefficient must be made constant in order to avoid linearly dependent solutions. Function $trunc()$ just eliminates the fractional part of the number.

The objective function to be the maximized is the product $G_c \times PPR$. Since the error surface has, in general, many local maxima, the results of the multivariable optimization are very dependent on the starting points. We have found strong relationship among the filters of different lengths that share the same core (i.e. the central coefficients are the same). This fact justifies using the process of eq. (2) to obtain good starting points for a multivariable optimization.

The method described above can be represented as a tree full of branches. Any time a filter is expanded, as in eq. (2), it can be either further expanded or it can undergo a multivariable optimization (keeping its size) before expanding it again. This process can be repeated indefinitely. Some times a branch can die (or stop) because, after a filter expansion, performance could not be improved. Also note that, due to the complex error surface, the results are affected by the order in which filter expansion and multivariable optimizations are performed. After going through most of branches up to size 14, and using many known filters as starting choices, some filters were chosen as examples and are presented in next section.

4 Results

Table 1 shows the lengths of the analysis lowpass and highpass filters (L_0/L_1), the coding gain (G_c), the peak-to-peak ratio (PPR), the $G_c \times PPR$ perceptive performance index and the regularity of wavelets designed and analyzed in this work. The power-signal noise ratio $PSNR$ obtained after image reconstruction is also listed. The optimizations were performed with a quasi Newton method using the BFGS formula for updating the approximation of the Hessian matrix. This was implemented using the MATLAB© optimization toolbox.

Wavelet W_1 is generated by the 5/3 filter bank reported as number 4 in [2] and A in [1]. Wavelet W_2 is the global optimum (maximum $G_c \times PPR$) for filter banks of length 6/6. Wavelet W_3 is obtained from the filter bank used to generate W_2 after undergoing the process in eq. (2) followed by a multivariable coefficient optimization. Wavelet W_4 is generated by the filter bank reported as number 3 in [2] and K in [1]. Wavelet W_5 is generated by the filter bank reported as D in [1]. Wavelet W_6 is generated from a multivariable optimization starting with the filter bank used to generate W_5. Wavelet W_7 is obtained from the filter bank used to generate W_2 undergoing the process in eq. (2) applied twice followed by a multivariable optimization. Wavelet W_8 is obtained starting with the filter bank used to generate the Haar wavelet followed by the process in eq. (2) applied 4 times, a multivariable optimization, the process in eq. (2) again and another multivariable optimization. Wavelet W_9 is generated by the filter bank F from [1]. Finally, wavelet W_{10} is generated from a multivariable optimization starting with the filter bank used to generate wavelet W_9.

From Table 1 it can be seen that wavelets W_2, W_3, W_6, W_7, W_8 and W_{10} show a very good value of $G_c \times PPR$, despite the orders of the prototype filters. It must also be noted that the small values of regularity of the analysis wavelets in all of the above mentioned cases do not mean a decrease in performance. This can be confirmed by the increase in coding gain when going from a 3 to a 5 stage decomposition. The effect of regularity will be felt only when the number of stages is very large, which is rarely the case in image processing. The value of the PPR is quite insensitive to the increase in the number of stages.

If Figure 1d the original 512X512 Lena image, sampled using 8bpp is displayed. In Figure 1a it is shown, in details, the region corresponding to Lena's face. In Figures 1b, 1c, 1e and 1f it can be observed the LENA image coded at 0.15bpp using the EZW algorithm [6], for wavelets W_1, W_2, W_4 and W_{10}, re-

Table 1: Characteristics of selected wavelets. Five stages used, except when noted.

wavelet	L_0/L_1	G_c (dB) (3 stg.)	G_c (dB) (5 stg.)	PPR	$G_c \times PPR$ (linear)	$reg(h_0)$	$reg(g_0)$	$PSNR$
W_1	5/3	9.35	9.59	1.60	13.780	0	1	30.50
W_2	6/6	9.34	9.67	1.94	16.666	-0.2	0.6	30.96
W_3	8/8	9.36	9.61	1.93	16.704	-0.3	1.2	30.83
W_4	6/10	9.59	9.88	1.65	15.055	0.7	2	31.28
W_5	10/10	9.51	9.66	1.60	14.325	1.2	2.3	30.86
W_6	10/10	9.32	9.56	1.94	16.598	-0.4	0.6	30.66
W_7	10/10	9.34	9.61	1.95	16.722	1.3	0.5	30.72
W_8	12/12	9.34	9.64	1.96	16.854	-0.3	1.3	30.87
W_9	10/14	9.61	9.61	1.59	14.478	1.3	2.8	31.17
W_{10}	14/14	9.41	9.62	1.94	16.970	-0.3	0.7	30.88

spectively, each with a 5 stage decomposition. In Figure 1b, the image was coded using wavelet W_1 and in Figure 1c, wavelet W_2 was used. The later produces an image with less blocking artifacts. This improvement is due, mostly, to the influence of G_c in the optimization. The filters of wavelet W_1 were designed by factorization of an interpolating polynomial, such as most of perfect reconstruction linear-phase filter banks. This example shows how the proposed process can improve on short filters. In Figure 1e, the image was coded using wavelet W_4 and in Figure 1f, wavelet W_{10} was used. Note that the amount of ringing is smaller when using wavelet W_{10}. This improvement can be attributed, mostly, to the role of the PPR in the optimization. Note that the filters of wavelet W_4 were also designed by factorization of an interpolating polynomial. This example shows how the proposed process can improve on long filters.

5 Conclusions

This work proposed a time-domain approach for filter design. It is different from both the usual methods of spectral factorization and lattice implementation, see [7]. It is also different from the time-domain technique proposed in [5], which had some limitations when designing a higher order filter from lower order prototype filter. Our approach is quite general, the only limitations being the problem of local maxima and the dependency of the results on the initial filter. This dependency can be minimized when using the method of increasing the size with one variable optimization and then performing a multivariable optimization.

From Table 1 it can be noted, as expected, that wavelets W_1, W_4, W_5 and W_9 show the worse perceptive results. This was so because these wavelets, found in the literature, were not optimized according the product $G_c \times PPR$. Note, however, that the values of the coding gain and the values of the actual $PSNR$ after image reconstruction are better for these wavelets (except for W_1) than for those we optimized, when the filter length is similar. Compare wavelets W_5 and W_6 and wavelets W_9 and W_{10}. This observation shows that perceptual improvement is achieved at the cost of objective performance. Perceptual criteria try to concentrate on errors which are hardly observed by the human eye.

By performing the optimization of the wavelets based on the product $G_c \times PPR$ as described herein, high performance wavelets for image coding can be designed with relatively low orders of the prototype filters. Also, when long filters are desired, in order to produce a better quality reconstruction, ringing can be kept very small.

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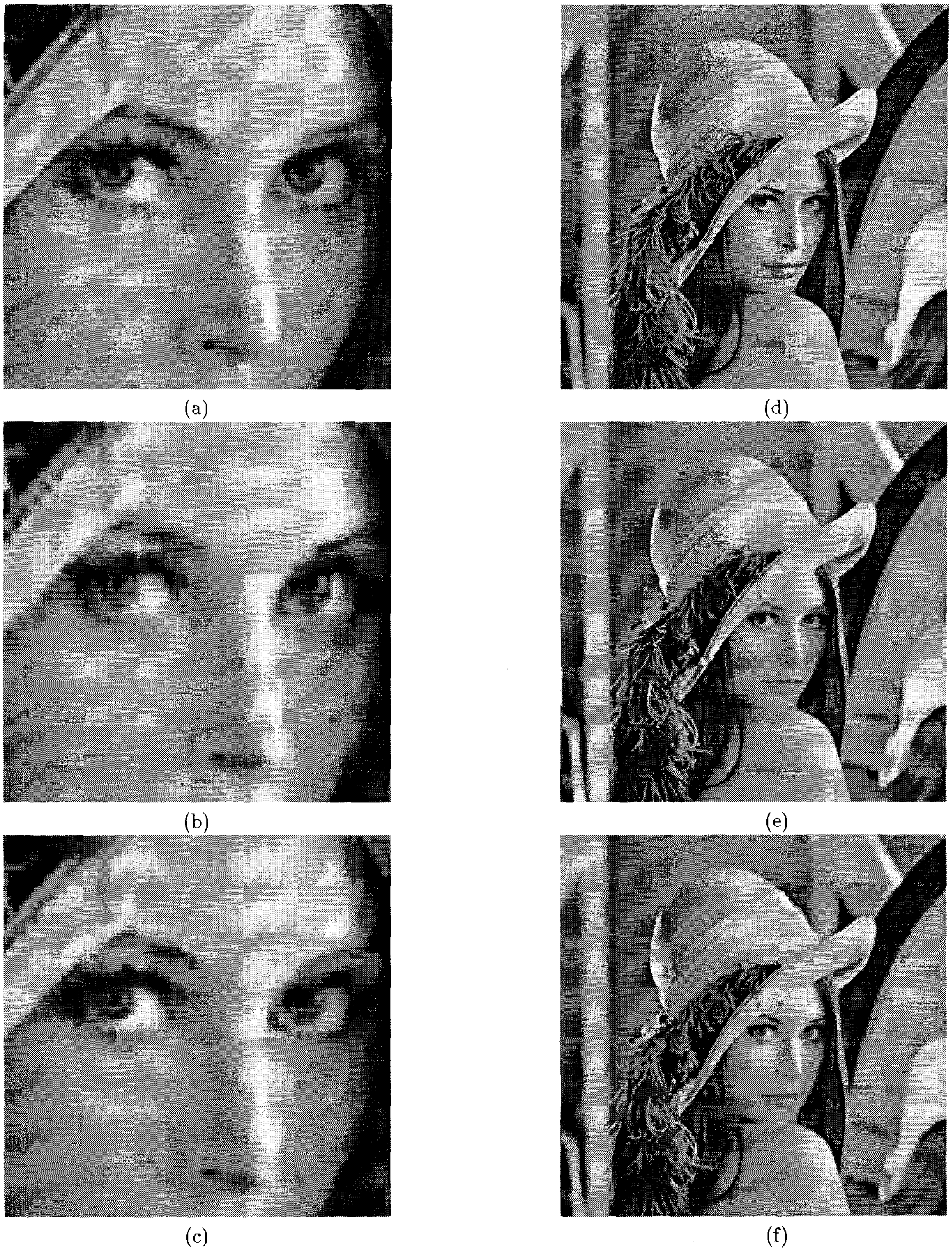

Figure 1: Lena image sampled at 8bpp: (a) Detail of face (128X128 pixels) and (d) Original (512 X 512 pixels). Coded imagens at 0.15bpp using wavelets: (b) W_1; (c) W_2; (e) W_4; and (f) W_{10}.