A Family of Wavelets for Image Compression Satisfying Perceptual Criteria

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Abstract

This paper presents a technique for biorthogonal linear-phase wavelet design for image compression. Wavelet design is based on direct optimization of the lowpass analysis filter using as objective the maximization of the function defined as the product of the theoretical coding gain and an index recently reported, called peak-to-peak ratio (PPR). With this function the perceptual quality expected for the compressed images can be evaluated without having to go through tests with experts. Also, such a measure can be used to design wavelets optimized according to perceptual criteria. A simplified design procedure was developed using a technique to generate high-order filters from lower-order ones. The filters thus obtained show a surprisingly good performance with relatively low orders.

1 Introduction

The construction of a wavelet decomposition scheme requires a suitable lowpass/highpass filter pair, or equivalently a two-band filter bank (FB). For image processing linear phase filters are preferred because of their low complexity [1] and to avoid artifacts produced by non-linear phase filters under quantization [2]. It is widely known that there are no paraunitary perfect reconstruction (PR) linear-phase solutions for two-channel FB except for trivial cases. Two-band FB yielding perfect reconstruction while keeping linear phase property, must be biorthogonal [3].

A number of results have been reported about good filter properties for image processing. In [4], it was verified that most of the objective performance criteria used in wavelet design, such as coding gain G_c, regularity of wavelets, number of vanishing moments and frequency selectivity have little correlation to subjective assessment of picture quality. In fact, it was noted that the shape of the synthesis wavelet plays an important role in the visibility of coding errors, specially how the wavelet amplitude decays towards its tails. A measure of such decay was defined, which involves the first 5 or 4 peaks (maxima or minima) of the wavelet, being called peak-to-peak ratio (PPR). The product ($G_c \times PPR$) was shown to have high correlation (≥ 0.85) to the subjective evaluation of the processed images by human experts.

In this paper, we study some interesting properties that arise when using the product ($G_c \times PPR$) to select the wavelet coefficients. Also a simple design technique starting from Haar filter banks resulted from our studies.

2 Filter design

Biorthogonality in a two-band FB is equivalent to

$$H_0(z)\,H_1(-z) - H_0(-z)\,H_1(z) = 2z^{-2m+1} \quad (1)$$

where $H_0(z)$ is the analysis lowpass filter of length L_0 and $H_1(z)$ is the analysis highpass filter of length L_1.

In most of the literature, both in the orthogonal [1] [5] and the biorthogonal [4] [6] cases, the design of a perfect reconstruction wavelet decomposition from iterated filters is done by optimization of the product polynomial

$$P(z) = H_0(z)\,H_1(-z) \quad (2)$$

followed by a suitable factorization, where the zeros of $P(z)$ are distributed between $H_0(z)$ and $H_1(z)$. This technique has some drawbacks such as:

- the resulting $H_0(z)$ and $H_1(z)$ are not necessarily optimal, even if $P(z)$ is optimized [7];
- the number of possible factors grows exponentially with respect to the order of filters [7];

- the factorization of $P(z)$ is numerically difficult when the number of zeros in $z = -1$ grows [5].

The problem with optimizing both $H_0(z)$ and $H_1(z)$ directly and simultaneously, instead of $P(z)$, is their highly non-linear relationship, as shown in eq. (1), what may create complex error surfaces, depending on the objective function used. Given a specific $H_0(z)$ there may be multiple functions $H_1(z)$ that follow eq. (1). In [3] a technique was proposed. It is given a known complementary filter pair $[H_0(z), H_1(z)]$, where the length of $H_1(z)$ is two less than the length of $H_0(z)$. While keeping $H_0(z)$ fixed a new polynomial $H_1(z)$ of higher degree than $H_0(z)$ can be found[1]. The increased degree of the polynomials allows for some optimization of $H_1(z)$. However, this result is of limited use because when keeping $H_0(z)$ fixed there is only one optimization parameter for each increase of four in the polynomial degree of $H_1(z)$. In this technique PR and linear-phase conditions are structurally imposed and the optimization is unconstrained.

It can be shown that there is only one filter $H_1(z)$ of order less or equal than the order of $H_0(z)$ ($L_1 \leq L_0$) [3,8] In this case, the relation (1) enables calculation of $H_1(z)$ from $H_0(z)$ by solving a set of linear equations, whenever there is a solution. This fails to happen when $H_0(z)$ has zeros of the type $z = \pm\alpha$ [3]. Since the synthesis lowpass filter $G_0(z)$, of length L_1, and the highpass filter $G_1(z)$, of length L_0, can be determined from the pair $(H_0(z), H_1(z))$ by

$$\begin{aligned} G_0(z) &= z^{2m-1} H_1(-z) \\ G_1(z) &= -z^{2m-1} H_0(-z) \end{aligned}$$

one can optimize the wavelet filter bank using any design criteria by simply changing the coefficients of $H_0(z)$. When it is desired that $L_1 \geq L_0$ the optimization can be performed by using $H_1(z)$ or $G_0(z)$ instead of $H_0(z)$. This approach is general because it spans the space of filters for which $L_1 = L_0 - 4k, k \in \mathbf{Z}$ for even L_0 and $L_1 = L_0 - 4k - 2, k \in \mathbf{Z}$ for odd L_0. Using an odd length $H_0(z)$ and an even length $H_1(z)$, or vice-versa, is known to yield filters of little practical interest [3].

In this work, the model of the input image signal will be assumed to be an auto-regressive process of order 1 (AR(1)), with autocorrelation $\rho = 0.95$. We will use the theoretical coding gain G_c from Katto and Yasuda [9].

The PPR measures the damping of the oscillations of the wavelet towards its tails. Such oscillations have direct influence on the visualization of quantization errors after image reconstruction. Increasing the damping (and consequently the PPR) will decrease the visibility of errors. An alternative to the measurement of the visibility of the coding errors which has often been considered in the literature is the filter length. However, it was verified in [4] to be poorly correlated to perceptual quality. It must be noted that PPR concerns the synthesis filters because it is involved in signal reconstruction.

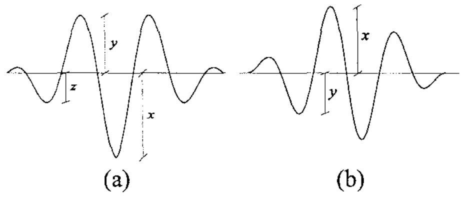

Figure 1: Parameters used in PPR calculation for: (a) symmetric wavelets; (b) anti-symmetric wavelets.

The PPR should be calculated from the synthesis wavelets; however, in practice, it is computed from an i stage iteration of the filter banks, where i chosen large enough for convergence.

According to Fig. 1, the PPR is given by eq. 3 for symmetric wavelets and by eq. (4) for anti-symmetric wavelets.

$$PPR = \frac{2(x+y)}{((x+y)+(y+z))} \tag{3}$$

$$PPR = \frac{2x}{x+y} \tag{4}$$

where x is the amplitude of the main peak of the wavelet, y is the amplitude of the second and z of the third peaks. The PPR can assume values in the interval $(0, 2]$, however considering that $x \geq y$, $x \geq z$ and $y \geq z$, this interval is reduced to $(1, 2]$ [4]. It was not defined for asymmetric wavelets.

3 Degenerated Filters

By observing the values of filter coefficients used for wavelet construction, it was noted that in many of them the ratio of the central coefficient to the first (or last) one was very high.

For example, consider the filter pair F in Table 1. This filter was calculated by factorization of $P(z)$, in eq. (2). $P(z)$ was optimized for maximum number of zeros at $z = -1$. It has the ratio of the central to the first coefficient equal to 135.8. This observation raises the question of whether the first and the last coefficients can be eliminated, and the order of the filter decreased by 2. For this particular filter the answer is affirmative, as seen on Table 2. After

[1] Also, if $H_1(z)$ has length greater than $H_0(z)$ solutions of lower degree for $H_1(z)$ could be found. Note that the length of filters must be odd. Even length solutions can be found if $P(z)$ polynomial is factored properly.

Table 1: Impulse response coefficients of even length filters used as examples. They are reported in [4] as filter F and filter D respectively. Only half of the filter coefficients are displayed.

	Filter F		Filter D	
	$H_0(z)$	$H_1(z)$	$H_0(z)$	$H_1(z)$
$h(0)$	0.028364	-0.0042364	0.026913	0.019844
$h(1)$	-0.015780	0.0023568	-0.032303	-0.023818
$h(2)$	-0.20170	0.025520	-0.24111	-0.023258
$h(3)$	0.063370	-0.0069033	0.054100	-0.14557
$h(4)$	0.83285	-0.051177	0.89950	0.54113
$h(5)$		-0.15720		
$h(6)$		0.57526		

Table 2: Performance of the wavelets generated from the filter pair F of Table 1 and its reduced length versions. In the product $G_c \times PPR$, G_c is not expressed in dBs. A 3 stage wavelet decomposition was used.

L_0/L_1	G_c (dB)	PPR	$G_c \times PPR$
10/14	9.60	1.59	14.48
12/12	9.53	1.70	15.30
10/10	9.57	1.66	15.03
8/8	9.03	1.90	15.17
6/6	8.98	1.91	15.09
4/4	8.32	2.00	13.59

the first order reduction the product $G_c \times PPR$ improves, what indicates that the original filter can be considered a degenerated case of the reduced length filter. Degenerated is used here in the sense that at least one degree of freedom is redundant (e.g., a straight line is a degenerated parabola). The new highpass analysis filter can be found by removing the first and last coefficients of $H_1(z)$. The new lowpass filter is found from eq. (1). Of course if $L_0 \geq L_1$, $H_0(z)$ would be used in replacement for $H_1(z)$.

The filters have an even length, what means that a zero at $z = -1$ is guaranteed and the filters can be candidates for wavelet construction. With filters of odd length the zero "moves away" from $z = -1$ in the process, so they will not yield regular wavelets.

By calculating the number of zeros of $P(z)$ (eq. 2) in the first two filters in Table 2, one could argue whether the 10/14 filter is a truly degenerated case of the filter 12/12, since $P(z)$, in both cases, has the same order. What will happen if the order reduction process is continued? Table 2 shows that wavelets generated by the reduced filters will also be good, up to length 6. The G_c is somewhat reduced, but the PPR compensates and keeps wavelet quality. In fact, PPR was used in [4] exactly because G_c alone has a poor correlation to the subjective grading set by human experts, after judging images processed using different wavelets.

An example of filter which is not degenerated, is filter D from Table 1, as can be seen in Table 3. The

Table 3: Performance of the wavelets generated from the filter pair D of Table 1 and its reduced length versions. In the product $G_c \times PPR$, G_c is not expressed in dBs. A 3 stage wavelet decomposition was used.

L_0/L_1	G_c(dB)	PPR	$G_c \times PPR$
10/10	9.51	1.60	14.33
8/8	0.03	1.55	1.55
6/6	6.52	1.53	6.87
4/4	7.03	1.72	8.70

ratio of the central coefficient to the first one is 33.4. However, for many filters, no correlation was found between this ratio and the possibility of reducing the size of the filter without affecting the performance of its corresponding wavelet.

4 Generating good wavelets from Haar filter banks

If the process used in section 3 is inverted, i.e., if one tries to degenerate (or, more precisely, increase the size of) small but good filters, what will happen? When reducing the size of a filter some information (a degree of freedom) is lost. Therefore, when increasing the size of the filter, it is possible to make some optimization, as shown below. We begin with the Haar biorthogonal filter pair $\left(\sqrt{2}\left(1+z^{-1}\right), \sqrt{2}\left(1-z^{-1}\right)\right)$. The performance of this wavelet is modest, see Table 4. However, after the first increase in size, we have the class of filters of length 4, for which $H_0(z)$ is:

$$\alpha\left[1+k\left(z^{-1}+z^{-2}\right)+z^{-3}\right] \tag{5}$$

where the α's are used herein as normalization factors.

The maximum of the product $(G_c \times PPR)$ is found when $k = -6.49$. Going further, we have the class of filters of length 6, for which $H_0(z)$ is:

$$\alpha'\left[1+k\left(z^{-1}-6.49z^{-2}-6.49z^{-3}+z^{-4}\right)+z^{-5}\right] \tag{6}$$

The maximum is found for $k = 9.6983$. Note that the process does **not** lead to optimum filters, as the best filter pair of length $L_0 = L_1 = 6$ (in the coefficient range $[-100, 100]$) is characterized by the following lowpass analysis filter:

$$\alpha''\left(1+2.25z^{-1}-33.4z^{-2}-33.4z^{-3}+2.25z^{-4}+z^{-5}\right) \tag{7}$$

However, both filters are very close, not only in the value of $G_c \times PPR$ (see Table 4), but also in the sense that, starting from the first one, the second filter is achieved by a simple gradient search algorithm over the surface of the function $G_c \times PPR$ (This can be shown by mapping the surface of $G_c \times PPR$ against the filter coefficients).

Table 4: Performance of the wavelets generated from Haar filters and its increased length versions. In the product $G_c \times PPR$, G_c is not expressed in dBs. A 3 stage wavelet decomposition was used. k is the optimized multiplier, as used in 5 and 6. The last filter is the one in equation (7)

Length	Opt. k	G_c(dB)	PPR	$G_c \times PPR$
2	——	7.92	2.00	12.45
4	-6.49	8.99	2.00	15.83
6	9.69	9.09	1.97	15.96
8	1.31	9.13	1.98	16.17
10	8.48	9.14	1.97	16.19
12	0.83	9.15	1.97	16.21
6 eq.(7)	——	9.34	1.94	16.67

Table 5: Performance of the wavelets generated from filters reported in the image processing literature. In the product $G_c \times PPR$, G_c is not expressed in dBs. A 3 stage wavelet decomposition was used.

filter	L_0/L_1	G_c(dB)	PPR	$G_c \times PPR$
K from [4]	6/10	9.59	1.65	15.06
C from [4]	9/7	9.46	1.48	13.04
B from [4]	9/3	9.38	1.49	12.90
2 from [6]	13/11	9.35	1.43	12.32
A from [4]	5/3	9.35	1.60	13.78
III from [2]	5/7	9.00	1.49	11.80
5 from [6]	2/6	9.18	1.79	14.84

Proceeding in the same way we can increase filter length by optimizing a single variable function. After some iterations (as the filter length increases), the variation in the index ($G_c \times PPR$) is very small, placing a limit in this process at length 8. However, there is good indication that we can use these filters to initialize a multi-variable optimization.

In general, regularity [10] of these filters is not very high. The lowpass analysis filters have regularity in the range $[-1, 0]$ while the regularity of the lowpass synthesis filters is in the range $[0, 1.2]$. Nevertheless, these values do not affect the performance of the wavelets, as shown in Table 4. In fact, using 4 or 5 stages in the wavelet decomposition increases the values of G_c, what confirms that regularity is not causing trouble.

The performance achieved with these filters is quite impressive and superior to most known wavelets, see Table 5. Note that Filter K in [4] is the same as filter 3 from [6]. Filter C in [4] is filter 1 in [6] and it is in Table II of [2]. Filter B in [4] is filter 6 in [6] which also appeared in Table I of [2]. Filter 2 in [6] is filter T in [4] but analysis and synthesis filters are exchanged. Filter A in [4] is filter 4 in [6].

5 Conclusions

In this work, we have used the product ($G_c \times PPR$) as an index to measure the quality of linear-phase filters for generating wavelets suitable for image processing. We verified that optimization can be done directly on the lowpass filter coefficients, instead of having to go through factorization of the product filter in eq. (2).

We found that, using the maximization of the performance index ($G_c \times PPR$) as a criteria, some reported filters can be regarded as degenerated versions of smaller ones.

A design technique was developed which yields good even length filters of higher order from low order ones. Surprisingly, we observed that they are superior in performance to most of the filters reported in the literature, even for relatively small filter lengths. Also, we noted that the increase in performance is asymptotic and it will not improve for lengths above 8.

References

[1] H. S. Malvar, *Signal Processing with Lapped Transforms*. Norwood, MA: Artech House, 1992.

[2] M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies, "Image coding using wavelet transform," *IEEE Trans. on Image Processing*, vol. 1, pp. 205–220, Apr. 1992.

[3] C. Herley and M. Vetterli, "Wavelets and filter banks: Theory and design," *IEEE Trans. on Signal Processing*, vol. 40, pp. 2207–2232, Sept. 1992.

[4] E. A. B. da Silva and M. Ghanbari, "On the performance of linear phase wavelet transforms in low bit-rate image coding," *IEEE Trans. on Image Processing*, vol. 5, pp. 689–704, May 1996.

[5] O. Rioul and P. Duhamel, "A remez exchange algorithm for orthonormal wavelets," *IEEE Trans. on Circuits and Systems II*, vol. 41, pp. 550–560, Aug. 1994.

[6] J. D. Villasenor, B. Belzer, and J. Liao, "Wavelet filter evaluation for image compression," *IEEE Trans. on Image Processing*, vol. 4, pp. 1053–1060, Aug. 1995.

[7] B.-R. Horng, *New Approaches to the Design of Two-Channel FIR Filter Banks with Application to Subband Image Coding*. PhD thesis, University of California, Los Angeles, USA, June 1990.

[8] T. Kailath, *Linear Systems*. System Sciences Series, Englewood Cliffs, NJ: Prentice-Hall, 1980.

[9] J. Katto and Y. Yasuda, "Performance evaluation of subband coding and optimization of its filter coefficients," in *Visual Communications and Image Processing '91*, pp. 95–106, SPIE, 1991.

[10] O. Rioul, "Simple regularity criteria for subdivision schemes," *SIAM Journal on Mathematical Analysis*, vol. 23, pp. 1544–1576, Nov. 1992.