A New Delayless Subband Adaptive Filter Structure

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Abstract—Adaptive subband techniques have been developed to reduce complexity and slow convergence problems of the traditional fullband high-order adaptive filters. Some of the disadvantages often encountered in most of the proposed architectures are the effect of aliasing associated with the multirate structure, which is a source of error in the modeling of the unknown system, and the delay introduced in the signal path. In this paper, we present a new delayless maximally decimated structure where the optimal subband filters are related to the wideband system in a closed form. They make use of a special DFT analysis filterbank where the polyphase components of the prototype filter represent fractional delays so that there is no need for adaptive cross-filters, and the unknown system can modeled perfectly in a closed-loop scheme. We interpret the proposed structure as a special case of a block adaptive filter with lower computational complexity than the conventional fullband LMS algorithm. Some computer simulations are presented in order to verify the good features of the proposed structure.

Index Terms—Adaptive filtering, multirate systems, subband systems.

I. INTRODUCTION

SUBBAND adaptive structures are very popular in applications where the unknown systems involved are characterized by long impulse responses, such as those encountered in acoustic echo cancellation. In this case, the subband filters are shorter, and fast convergence is achieved in each channel when using a gradient-based algorithm [1]. Fig. 1 illustrates the conventional subband scheme, where the error signal is locally evaluated in each subband. It was shown in [4] that for correct modeling of the unknown system \( G(z) \), the reconstruction filterbank along with adaptive cross-filters among subbands are necessary in an open-loop configuration. This requirement increases the computational burden, and the algorithm suffers from slow convergence in comparison with the fullband scheme. In a closed-loop structure, a delay is introduced in the error feedback path, which makes the convergence much slower. In [2], this problem was addressed, and a delayless architecture was proposed where the adaptive weights are computed in subbands and collectively transformed to an equivalent fullband filter. Another work addressing this idea can be found in [3], where the fullband coefficients are formed without frequency transformation. In this paper, we use a similar idea, but the wideband system is recovered through a different procedure. The necessity of adaptive cross-filters is eliminated by proper choice of the analysis filterbank, where in this case, the polyphase components of the resulting prototype filter \( H_0(z) \) represent fractional delays (FD). The FD implementation can be simple. It can also be verified that an open-loop scheme generates an excess of mean square error (MSE) because in such a scheme, we are actually minimizing the subband error energy. A closed-loop scheme is proposed, which is best suited for applications where low MSE is required, whose cost function is based on the fullband error signal, i.e., the fullband error signal is decomposed in subbands to update the subband weights.

In Section II, we derive an expression for the optimal adaptive structure \( W(z) \) in Fig. 1 based on polyphase domain analysis, where a maximally decimated DFT analysis filterbank is designed to avoid the cross-terms of \( W(z) \). In Section III, a subband/wideband filter mapping is described, based on a simple relation derived in Section II, and the adaptation algorithms for the proposed open-loop and closed-loop schemes are presented. The excess MSE for the new structure is estimated in Section IV, whereas the computational complexity is calculated in Section V. The theoretical results are verified via computer simulations in Section VI. Finally, in the Appendix, we establish a connection between the proposed structure and a recent work on block adaptive filtering.

Regarding notation, we used boldface for vectors and matrices.
In Fig. 1, consider the following. 

- \( G(z) \) unknown system to be identified; 
- \( W(z) \) adaptive filter matrix; 
- \( x(n) \) input signal; 
- \( d(n) \) desired signal. 

We define the \( z \)-transforms of the blocked versions of \( x(n) \) and \( d(n) \) as 

\[
X(z) = \sum_{n} x(n)z^{-n} \tag{1}
\]

\[
D(z) = \sum_{n} d(n)z^{-n} \tag{2}
\]

where 

\[
x(n) = \begin{bmatrix}
x(Mn) \\
x(Mn-1) \\
\vdots \\
x(Mn-M+1)
\end{bmatrix} \tag{3}
\]

\[
d(n) = \begin{bmatrix}
d(Mn) \\
d(Mn+1) \\
\vdots \\
d(Mn-M+1)
\end{bmatrix} \tag{4}
\]

The analysis filter transfer functions \( H_i(z), \ i = 0, \ldots, M-1 \), can be expressed in terms of their polyphase components as \( [5] \)

\[
H_i(z) = \sum_{k=0}^{M-1} z^{-k}E_d(z^M) \tag{5}
\]

as illustrated in Fig. 2.

Defining the polyphase matrix \( E(z) = [E_d(z)] \) and using the noble identities \( [5] \), the subband signals can be written in vector form as 

\[
Y(z) = W(z)E(z)X(z) \tag{6}
\]

\[
V(z) = E(z)D(z) \tag{7}
\]

where \( V(z) \) is the desired signal split in subbands, and \( Y(z) \) is the adaptive system output, as shown in Fig. 1.

In order to find a closed-form expression for the optimal \( W(z) \), we need to represent \( G(z) \) in its equivalent blocked system. From multirate theory, it is well-known that a scalar linear time-invariant transfer function \( G(z) \) can be implemented through a multirate system as a pseudocirculant matrix \( G(z) \) by means of a blocking mechanism \( [5] \). For example, a \( 3 \times 3 \) matrix \( G(z) \) is given by 

\[
G(z) = \begin{bmatrix}
G_0(z) & G_1(z) & G_2(z) \\
G_2(z) & G_0(z) & G_1(z) \\
G_1(z) & G_2(z) & G_0(z)
\end{bmatrix} \tag{8}
\]

where \( G_m(z), m = 0, 1, 2, \) are the polyphase components of \( G(z) \). Fig. 3 illustrates the equivalent multirate structure of Fig. 1.

We note that the cascade of the unblock/block mechanisms in the dashed box cancels out and the blocked desired signal is given simply by 

\[
D(z) = G(z)X(z) \tag{9}
\]

and from (7), we have 

\[
V(z) = E(z)G(z)X(z). \tag{10}
\]

Now, setting the channel error vector \( E(z) = V(z) - Y(z) \) to zero, if \( X(z) \neq 0 \), we have 

\[
E(z)G(z) = W_o(z)E(z) \tag{11}
\]

or 

\[
W_o(z) = E(z)G(z)E^{-1}(z) \tag{12}
\]

where \( W_o(z) \) is the optimal solution for the adaptive filter matrix. This expression is similar to the one derived by Gilloire and Vetterli in \( [4] \) for the modulation domain analysis, and since \( W(z) \) is nondiagonal, it requires cross-filters among channels in order to model the unknown system correctly. The framework presented thus far is somewhat related to a recently proposed block adaptive filtering, as discussed in more detail in the Appendix.

From (12), we can infer that the off-diagonal terms of \( W_o(z) \) can be eliminated simply by choosing the matrix \( E(z) \) as a similarity transformation that turns the pseudocirculant matrix \( G(z) \) into its Jordan form. It can be easily verified \( [6] \) that the eigenvectors of an \( M \times M \) pseudocirculant matrix are given by the columns of 

\[
Q(z) = \Lambda(z)F \tag{13}
\]

where 

\[
\Lambda(z) = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & z^{-1/M} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & z^{-(M-1)/M}
\end{bmatrix} \tag{14}
\]

and \( F \) is the \( M \times M \) DFT matrix with \( F = [W^{km}] \), where \( W = e^{-j2\pi/M} \) is the \( M \)th root of unity. The optimal adaptive filters are then given by the eigenvalues of \( G(z) \), i.e., 

\[
W_o(z) = \sum_{k=0}^{M-1} G_k(z)z^{-k/M}W^{ki} \tag{15}
\]
Therefore, the resulting prototype $H_0(z)$ will be a length-$M(N_{FD}+1)$ FIR filter. For even-length FD filters, the impulse response of the filter associated with the fractional delay $d_i = -i/M$ is the same as the one associated with $d_i = (M-i)/M$ in reverse order. In this case, the prototype filter has linear phase. Besides, the prototype filter must be a Nyquist ($M$) filter because the prototype filter has one polyphase component equal to unity, as we can see in Fig. 4.

Actually, it can be shown that one technique for approximation of a fractional delay is to design a symmetric $M$th-band filter and pick its $i$th polyphase component to represent the delay $D_{int} + i/M$ [7]. Then, we conclude that in order to eliminate adaptive cross-filters, a DFT filterbank with lowpass prototype filter given by an $M$th-band filter should be used. We verified in our simulations that good results were obtained when using a Nyquist ($M$) prototype filter designed through the eigenfilter method [8]. This approach based on fractional delays will be useful in understanding the proposed structure described next.

III. PROPOSED STRUCTURE

In a subband system, the signal reconstruction introduces a delay in the signal path, which can be avoided by mapping the subband adaptive filters into a wideband filter. Equation (15) describes the form of the optimal adaptive filters in each subband. They represent the eigenvalues of a pseudocirculant matrix. Note that these eigenvalues are indeed the DFT of the first row of the circulant matrix shown in (18) at the bottom of the next page.

Therefore, taking the IDFT of the adaptive filters, we can recover the polyphase components of the unknown system but weighted by fractional delays, i.e.,

$$
\begin{bmatrix}
\hat{G}_0(z) \\
\hat{G}_1(z)z^{-(1/M)} \\
\vdots \\
\hat{G}_{M-1}(z)z^{-(M-1)/M}
\end{bmatrix} = \text{IDFT} \begin{bmatrix}
W_0(z) \\
W_1(z) \\
\vdots \\
W_{M-1}(z)
\end{bmatrix}. 
$$

(19)

From (15), note that the length of the adaptive filters $W_i(z)$ must agree not only with the polyphase component length...
(which is equal to $N/M$, where $N$ is the unknown system length) but should also be high enough to model the fractional delays. Actually, we verified that an increase of one sample is sufficient to model the unknown system perfectly in a closed-loop scheme. Since any filter $G_i(z)$ has an inherent fractional delay, it is reasonable that the product $G_i(z)z^{-i/M}$ represents a filter with one more sample than $G_i(z)$. We therefore use adaptive filters $W_i(z)$ of lengths $L = N/M + 1$. Representing each $W_i(z)$ by an $L$-length vector $\mathbf{w}_i(n)$, we have to compute $L$-IFFT’s of length $M$ to recover the weighted polyphase components, i.e., defining $\mathbf{w}(n)$, the $L$-length vector representing the quantity $W_i(z) = \hat{G}_i(z)z^{-i/M}$, we compute

$$
\begin{bmatrix}
  u_0^i(k(n)) \\
  u_1^i(k(n)) \\
  \vdots \\
  u_{M-1}^i(k(n))
\end{bmatrix} = \text{IDFT}\left( 
\begin{bmatrix}
  u_0(k(n)) \\
  u_1(k(n)) \\
  \vdots \\
  u_{M-1}(k(n))
\end{bmatrix}
\right)
$$

for $k = 0, 1, \ldots, L - 1$. In order to recover the polyphase components $\hat{G}_i(z)$ from $W_i(z)$, note that

$$\hat{G}_i(z)z^{-i/M}z^{-(M-i)/M} = \hat{G}_i(z)z^{-i}, \quad i = 1, \ldots, M - 1.$$  

This means that convolution of $\mathbf{w}_i(n)$ with the impulse response of the FD filter $E_{i-1}(z)$, $i = 1, \ldots, M - 1$ will produce the polyphase component $\hat{G}_i(z)$ delayed by $D_{int} + 1$ samples.

Note that impulse response of $\hat{G}_0(z)$ is readily represented by $\mathbf{w}_0(n)$. Defining $\hat{g}_i(n)$ as the $N/M$-length vector corresponding to the $i$th polyphase component, we have

$$W_i(z) \approx \hat{G}_i(z)$$

$$W_i(z)E_{i-1}(z) \approx \hat{G}_i(z)z^{-(D_{int}+1)}$$

for $i = 1, \ldots, M - 1$. In other words, for the first polyphase filter $\hat{G}_0(z)$, we simply discard the last sample of $\mathbf{w}_0(n)$. For $\hat{G}_i(z)$ $i = 1, \ldots, M - 1$, we discard the first $D_{int} + 1$ samples and retain the next $L - 1$ ones. Finally, the fullband filter is formed by

$$\hat{G}(z) = \sum_{i=0}^{M-1} \hat{G}_i(z)z^{-i}.$$  

Fig. 5 illustrates the above procedure in an open-loop configuration. From the construction of Fig. 3, we see that the adaptation algorithm can be derived by minimizing the following open-loop cost function based on the subband error signals

$$e_{op} = \sum_{i=0}^{M-1} E[\epsilon_i(n)]^2.$$  

Thus, for the above objective function, it can be easily shown that a normalized LMS adaptation algorithm (NLMS) for each channel is given by

$$\epsilon_i(n) = v_i(n) - \mathbf{w}_i^T(n)\mathbf{u}_i(n)$$

$$w_i(n+1) = w_i(n) + \frac{2\mu}{\sigma_i^2(n)} \mathbf{u}_i(n)\epsilon_i(n)$$

$$\sigma_i^2(n) = \beta\sigma_i^2(n-1) + (1 - \beta)|u_i(n)|^2$$

where

$$\epsilon_i(n)$$ error signal locally evaluated in the $i$th subband;

$\mathbf{u}_i(n)$ input to the $i$th adaptive filter;

$u_i(n)$ first element of $\mathbf{u}_i(n)$.

The recursion (27), with $0 < \beta < 1$, estimates the power of $\mathbf{u}_i(n)$, assuming that the channel signals are stationary. The range of values for the convergence factor is typically

$$0 < \mu < \frac{1}{L}.$$  

We can find tighter upper bounds for the convergence factor depending on the statistics of the input signal, but these are used less often in practice.

Since the filterbanks are not ideal, the minimization of (24) will not necessarily reduce the fullband error energy to a minimum MSE, i.e., the unknown system may not be identified accurately. A natural way to solve this problem is to use the fullband error signal split in subbands to update the adaptive filters, leading to a closed-loop scheme. Fig. 6 shows the detailed structure proposed for the closed-loop configuration. In this case, the NLMS updating equation is given by

$$w_i(n+1) = w_i(n) + \frac{2\mu}{\sigma_i^2(n)} \mathbf{u}_i(n)\epsilon_i(n)$$

where the fullband error is evaluated as

$$e(n) = d(n) - \hat{g}_i^T(n)x(n)$$

and split in subbands generating $\epsilon_i(n)$, as depicted in Fig. 6. A closed-loop scheme allows the minimization of a cost function based on the fullband error signal and guarantees that the algorithm converges to a minimum MSE.

IV. EXCESS MSE

In this section, we analyze the excess MSE due to gradient noise, which goes to zero as $\mu \to 0$, and does not refer to the additional error that results from nonideal filterbanks, as just discussed in the last section. We will estimate the excess MSE only for the closed-loop scheme. The expression for the excess MSE for the open-loop scheme will not be presented since it can be derived in a similar manner.

Now, let us return to Fig. 6. We can slightly modify the subband/wideband transformation so that it can be interpreted
as a synthesis of the subband adaptive filters through a filterbank, as illustrated in Fig. 7. Now, define the polyphase matrix associated with the synthesis filterbank of Fig. 7 as

$$P(z) = \sum_{n=0}^{N_{FB}} P(n)z^{-n}$$

(31)

where $P(n)$ is the time-domain matrix related to $P(z)$. In addition, define $T$ as the $N \times (N+M)$ matrix

$$T \triangleq \begin{bmatrix} P(D') + 1 & \ldots & P(0) & 0 & 0 & \ldots & \ldots \\ P(D' + 2) & \ldots & P(1) & P(0) & 0 & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ P(D' + L) & \ldots & P(L - 1) & \ldots & P(0) & \ldots & \ldots \\ 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{bmatrix}.$$ 

(32)

The wideband filter $\tilde{g}(n)$ can then be expressed as

$$\tilde{g}(n) = TBW(n) = AW(n)$$

(33)

where

$$W(n) \triangleq \begin{bmatrix} w_0(n) \\ w_1(n) \\ \vdots \\ w_{M-1}(n) \end{bmatrix}.$$ 

(34)

$A = TB$, and $B$ is a $(N+M) \times (N+M)$ matrix with one and zero entries that places the adaptive filters in an appropriate form to the matrix multiplication. For a $9 \times 9$ example, it gives

$$B \triangleq \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$ 

(35)

The discarding-of-samples step described in Section III, is eliminated by starting the convolution with the polyphase...
filters after \( D_{\text{int}} + 1 \) samples, as can be observed from the first row of the matrix \( \mathbf{T} \).

Due to gradient estimation noise, there is an error in the internal signals \( \hat{\mathbf{g}}(n) \) that is denoted as \( \Delta \hat{\mathbf{g}}(n) \). The fullband excess MSE can be expressed as [9]

\[
J_{\text{exc}}(n) = \text{tr}\{\mathbf{R} \mathbf{K}(n)\} \tag{36}
\]

where \( \mathbf{R} \) is the input correlation matrix, and \( \mathbf{K}(n) = E[\Delta \hat{\mathbf{g}}(n)\Delta \hat{\mathbf{g}}^H(n)] \) is the covariance matrix of \( \Delta \hat{\mathbf{g}}(n) \). Using the relation in (33), the excess MSE can be written as

\[
J_{\text{exc}}(n) = \text{tr}\{\mathbf{R} E[\Delta \hat{\mathbf{g}}(n)\Delta \hat{\mathbf{g}}^H(n)]\} = \text{tr}\{\mathbf{R} \mathbf{A} E[\Delta \mathbf{W}(n)\Delta \mathbf{W}^H(n)] \mathbf{A}^H\} = \text{tr}\{\mathbf{A}^H \mathbf{R} \mathbf{A} E[\Delta \mathbf{W}(n)\Delta \mathbf{W}^H(n)]\} = \text{tr}\{E[\mathbf{A}^H \mathbf{x}(n)\mathbf{x}^H(n)\mathbf{A}] E[\Delta \mathbf{W}(n)\Delta \mathbf{W}^H(n)]\} \tag{37}
\]

where \( \Delta \mathbf{W}(n) \) is the deviation in the subband adaptive coefficients due to gradient estimation error, and

\[
\mathbf{x}(n) \triangleq \begin{bmatrix} x(n) \\ x(n-1) \\ \vdots \\ x(n-L+1) \end{bmatrix} \tag{38}
\]

where \( x(n) \) is given by (3). Note that the product \( \mathbf{A}^H \mathbf{x}(n) \) represents the transpose of the synthesis filterbank operation.
applied to the input signal $X(n)$, i.e., it represents an analysis operation applied to $x(n)$. Observing the polyphase structure of this synthesis filterbank, we see that its corresponding prototype filter is practically the same as the one of the analysis filterbank, as can be seen from their impulse responses shown in Fig. 8 (the only difference is that the last polyphase component now comes first). Hence, considering that the subband signals and the adaptive coefficients are uncorrelated, this expression reduces to

$$ J_{exc}(n) = \sum_{i=0}^{M-1} \text{tr}\{E_i[n] u_i(n) u_i^H(n) K_i(n)\} $$

$$ = \sum_{i=0}^{M-1} J_{exc_i}(n) $$

where the overall excess MSE is given by the sum of the excess MSE’s $J_{exc_i}(n)$ in the subbands. Considering that we have a prototype with good attenuation, this expression can be approximated as [9]

$$ J_{exc}(n) \approx \sum_{i=0}^{M-1} \frac{\mu_i}{1 - \mu_i} \text{tr}[\text{R}_i] $$

where

$$ \sigma^2_{m_i} \approx \sigma^2_{m_i} / M $$

and $\text{R}_i = E_i[n] u_i(n) u_i^H(n)$. In this case, for a white noise input with unit variance and using the same convergence factor $\mu$ for all the subbands, this expression can be simplified to

$$ J_{exc}(n) = \frac{\mu \sigma^2_{m} L}{1 - \mu L} $$

where $L$ is the length of the adaptive filters.

V. COMPUTATIONAL COMPLEXITY

The complexity required by the new structure involves the computation of

i) the filterbank decomposition;

ii) the subband NLMS algorithm;

iii) the wideband filter convolution;

iv) the recovering of the wideband filter from the subband filters.

Here, the computational complexity will be given in multiples per input sample, considering that the product of complex values is implemented through four real multiplies.

The subband decomposition requires one convolution of a $K$-length prototype filter and one $M$-point FFT for each block of $M$ input samples. Thus, the subband splitting requires

$$ C_1 = 2K/M + 2 \log_2 M $$

real multiplies per input sample for the two analysis filterbanks. Because of the symmetry of the IFFT for real signals, we only have to process half of the $M$ channel complex signals. Therefore, we have to update $M/2$ adaptive filters of length $L = N/M + 1$ for every $M$ input samples. Thus, the subband processing requires

$$ C_2 = 2N/M + 2 $$

real multiplies per input sample. For the open-loop scheme, we have to evaluate the adaptive filters outputs, which requires an additional $C_2$ real multiplies per input sample.

For the subband/wideband adaptive filter mapping, we have to compute $L$ IFFT’s for the transformation $w_i(n) \rightarrow w_i^T(n)$ and perform $M - 1$ convolutions with the polyphase filters. In practice, it is not necessary to evaluate the wideband filter every $M$ input samples because its output cannot vary much faster than the length of its impulse response. The same idea was used in [2], and we only need to perform the subband/wideband mapping for $M/2$ input samples. The convergence behavior for different values of $I$ will be examined in the simulations. In this case, we will show that by increasing $I$, we can maintain the convergence rate and reduce the computational complexity of this part. Since we need to evaluate $N/M$ samples at the output of the $M - 1$ polyphase filters $E_i(z)$, the computational complexity of the convolution is $(M - 1)(N/M)$, i.e., the polyphase filters are shorter than the adaptive filters, this part requires

$$ C_3 = \left( (N/M + 1) \log_2 M + \frac{NK(M - 1)}{M^3} \right) \frac{1}{I} $$

where $L$ is the length of the adaptive filters. A further reduction in the computational complexity could be achieved by implementing the synthesis operation by using the Nussbaumer polynomial transform [10], but this is a more specific subject and will not be discussed here.

The wideband convolution can be performed in the same way described in [2] by partitioning the wideband filter in $p$ segments and using fast convolution techniques [11], [12]. In this case, the number of multiplies per input sample is given by

$$ C_4 = N/p + 2(p + 1) \log_2(2N/p) + 4(p - 1) $$

The number of segments $p$ can also be optimized so that the complexity over the direct convolution is minimized.
Therefore, the total computational complexity for the closed-loop scheme is

$$C_{cl} = C_1 + C_2 + C_3 + C_4$$

(48)

whereas for the open-loop scheme, we have

$$C_{op} = C_1 + 2C_2 + C_3 + C_4.$$  

(49)

In order to compare with the example presented in [2], consider $N = 512$ taps, $M = 32$ subbands, and a $K = 128$ tap prototype filter. In this case, (47) is optimized with $p = 6$. For the structure of [2], when the wideband transformation was performed for each $N$ input samples, the total computational load consists of 401 multiplies per input sample in the case of the closed-loop configuration. This corresponds to a reduction by a factor of 2.5 in the computational complexity of the fullband LMS. The open-loop scheme required 502 multiplies per input sample, which corresponds to a reduction of 2.0. Note that the filterbank used in [2] is oversampled by $M/2$. Whereas here, we use a maximally decimated structure. For the proposed closed-loop structure, calculating the subband/wideband mapping for each $N$ input samples corresponds to using $I = 16$, and it gives $C_1 = 18$, $C_2 = 34$, $C_3 = 9$, and $C_4 = 218$, with the total of 279 real multiplies per input sample. For the open-loop version, we have one additional $C_2$, which requires 313 real multiplies per input sample. We see that the complexity over the fullband LMS is reduced by a factor of 3.7 for the closed-loop version and 3.5 for the open loop. Using $I = 1$ gives $C_3 = 147$, and the total complexity of 417 multiplies/sample is comparable with the one proposed in [2]. Fig. 9 illustrates the computational complexity reduction of the proposed structure in comparison with the fullband scheme by varying the number of subbands $M$ and the length of the unknown system. The number of segments $p$ is always adjusted so that $C_4$ is optimized.

VI. COMPUTER SIMULATIONS

We now verify the performance of the new structure via simulations. For all the simulations, a white noise of variance $-100 \text{ dB}$ was added to the desired signal. The convergence factor $\mu$ was chosen so that a maximum convergence rate was achieved in any simulation while still preserving stability.

In order to verify the effect of the length of the adaptive filters in the modeling of the unknown system, we compared the convergence of the algorithm for $N/M$ and $N/M + 1$ length adaptive filters. Fig. 10 shows this comparison (white noise input) for an $N = 64$ tap unknown system and an $M = 8$-channel filterbank with a 64-tap prototype filter ($I = 1$).

Although it has fast convergence, the open-loop scheme presents high MSE since the fullband error signal is not minimized. Good performance is then achieved by the closed-loop scheme, where $N/M + 1$ length adaptive filters are sufficient to model the unknown system perfectly.

Now we compare the performance of the new structure in the closed-loop configuration with the fullband NLMS. For an $N = 256$-tap unknown system, an $M = 16$-channel filterbank was designed, resulting in $N/M + 1 = 17$-tap adaptive filters. Fig. 11 shows the convergence for a white noise input. The convergence factor $\mu$ was 0.025 for the new structure and 0.0156 for the fullband system. The performances of both structures are similar since the filterbank does not affect the eigenvalue spread of the input signal.

The advantage of implementing the subband structure becomes apparent for colored input signals. Figs. 12 and 13 show the MSE decay for a first-order AR process with a real pole at 0.9 (using a 16-channel filterbank) and a real speech input signal sampled at 8 kHz (using an eight-channel filterbank), respectively. In the former, we used the same convergence factors of the previous example, and in the latter, we used $\mu = 0.013$ for the proposed structure. In this case, fast convergence is achieved in comparison with the fullband scheme.

For the same example used in Fig. 10, using the closed-loop scheme, we can observe the effect of increasing the factor $I$ on the convergence behavior in Fig. 14. For $I$ ranging from 1 to 5, the convergence rate does not change significantly. For $I = 8$, we are actually updating the wideband filter each $N$ input samples. In conclusion, we can reduce the computational complexity by using higher values of $I$ without reducing the
convergence speed to the level of the fullband LMS for colored inputs.

In order to verify the theoretical estimate of the MSE, we simulated the proposed structure using an additive noise of variance $-40$ dB at the output with colored noise input signal. Table I shows the results of some experiments for different values of the number of subbands $M$ and the prototype length $K$. We note that these results are in good agreement with (43), provided that we have designed a good prototype filter or a filterbank with sufficient number of subbands.

VII. CONCLUSIONS

We proposed a new structure with the following features.

1) It is maximally decimated.

2) It eliminates the necessity of adaptive cross-filters since the fullband error feedback compensates for any excess MSE due to aliasing.

3) No delay is introduced in the signal path.

4) The analysis filters consist of a uniform DFT filter that tries to cancel the aliasing components in the subbands.

We described an exact relation between the wideband coefficients and the adaptive subband filters. For the closed-loop configuration, the unknown system can be modeled perfectly. We have shown that the new structure can reduce by 3.7 times the computational complexity of the fullband LMS and still improves the convergence rate of the algorithm in comparison with the fullband scheme.

APPENDIX

In this Appendix, we relate adaptive filtering to previous work on block adaptive filtering.
Consider a linear time-invariant system $g(n)$ and its Wiener solution resulting from the minimization of the error signal energy $E[|e(n)|^2]$. A corresponding multichannel Wiener solution can be derived by blocking the scalar input-output signal description, as illustrated in Fig. 15.

In this case, for jointly-WSS signals $x(n)$ and $d(n)$, the optimal matrix filter obtained when we minimize the cost function

$$E[e^H(n)e(n)]$$

is given by [13]

$$G_o(z) = \hat{S}_{xx}(z)S_{xx}^{-1}(z)$$

where $S_{xx}(z)$ and $\hat{S}_{xx}(z)$ are the $z$-transform of the auto-correlation $R_{xx}(k) = E[x(n)x^H(n-k)]$ and crosscorrelation $R_{xg}(k) = E[x(n)g^H(n-k)]$, respectively. The tilde operator denotes transpose, replacement of $z$ by $z^{-1}$, and complex conjugate of the coefficients of $S_{xx}(z)$.

Now, consider a scalar LTI system $g(n)$ of length $N$. Clearly, the pseudocirculant $G(z)$ obtained by blocking $g(n)$ will be a matrix polynomial of length $L = N/M + 1$. Hence, we can express this filter in the time domain as an $(N + M) \times M$ matrix

$$G(n) = [G_0(n) \ G_1(n) \ \cdots \ G_{L-1}(n)]^T$$

(52)

where $G_i(n) i = 0, \cdots, L - 1$ are $M \times M$ blocks. Now, let the input vector for $G(n)$ be $X(n)$ defined in (38). Then, the multichannel correlation and crosscorrelation matrices for the matrix adaptive filter can be written as

$$R = \begin{bmatrix}
R(0) & R(1) & \cdots & R(L - 1) \\
R(1) & R(0) & \cdots & R(L - 2) \\
\vdots & \vdots & \ddots & \vdots \\
R(L - 1) & R(L - 2) & \cdots & R(0)
\end{bmatrix}$$

(53)

$$R = E[X(n)X^H(n)]$$

(54)
From the above block formulation, we can define the block LMS algorithm whose updating equation is

$$G(n+1) = G(n) + 2\mu \mathcal{X}(n) \mathbf{e}^H(n).$$

(57)

The block LMS was proposed by Sathe and Vaidyanathan for identification of bandlimited channels [13]. In this reference, it is shown that for this type of application, the input to the adaptive filter is in general cyclostationary. A stochastic process $x(n)$ is said to be cyclo-WSS with period $M$ if $[\text{CWSS}_M]$ if

$$E[x(n)] = E[x(n + kM)], \quad \forall n, \forall k$$

$$r_{xx}(n, k) = r_{xx}(n + M, k), \quad \forall n, \forall k.$$  

(58)

Thus, the use of a scalar adaptive filter for a (CWSS)$_M$ input will not be optimal in terms of a Wiener solution because this solution applies for stationary signals. Since the blocking version of a (CWSS)$_M$ input is a WSS vector, a matrix updating equation defined by (57) is best suited for that type of application. The problem concerning this scheme is that convolution and updating of a $(N + M) \times M$ matrix must be performed, making its computational complexity so high that its usefulness is limited.

The above computational burden can be reduced by decomposing the matrix $G(z)$, as in (12). Fig. 16(a) illustrates this procedure, where the transformed version of the blocked error $e(n)$, say $e'(n)$, is used to update the matrix coefficients. Since $E(z) = Q^{-1}(z)$, this scheme can be modified equivalently to Fig. 16(b), where we still preserve the fullband error information. In fact, this scheme represents the conventional closed-loop subband structure, where the channel signals are reconstructed so that we can evaluate the error signal in fullband. We can go further and, instead of using a conventional subband structure, can use a delayless closed-loop scheme for this application.

Note that the original matrix adaptive filter is a matrix polynomial of length $N/M + 1$. This is in good agreement with the fact that the adaptive filters in each subband should be able to represent the unknown system $(N/M$ length polyphase components) plus one sample split in fractional delays when we perform the diagonalization of $G(z)$.

The closed-loop algorithm (29) is updated by using the transformed error vector $e'(n)$, i.e., clearly, it minimizes the cost function

$$\xi_{eA} = E[\mathbf{e}^H(n) \mathbf{e}'(n)].$$

(59)

Let $S_{ee}(e^{j\omega})$ and $S_{ee}(e^{j\omega})$ be the power density spectra of the vector processes $e(n)$ and $e'(n)$. The above equation can be written as

$$\xi_{eA} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{tr} [S_{ee} e'(e^{j\omega}) \mathbf{e} H(e^{j\omega})] d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{tr} [E(e^{j\omega}) S_{ee} e'(e^{j\omega}) \mathbf{e} H(e^{j\omega})] d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{tr} [\mathbf{e} H(e^{j\omega}) E(e^{j\omega}) S_{ee}] d\omega.$$  

(60)

Since $\mathbf{e} H(e^{j\omega}) E(e^{j\omega}) = I$, the term inside the trace becomes

$$\xi_{eA} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{\omega=0}^{M-1} S_{ee}(e^{j\omega}) d\omega$$

$$= \sum_{\omega=0}^{M-1} E[|\mathbf{e}(n)|^2]$$

$$= E[|\mathbf{e}(n)|^2].$$  

(61)

which is (50). In particular, for stationary signals, we have

$$\xi_{eA} = E[|\mathbf{e}(n)|^2].$$  

(62)

REFERENCES


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